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Reliability-based design optimization of structural systems under hybrid probabilistic and interval model

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ABSTRACT

Reliability-based design optimization (RBDO) under hybrid probabilistic and interval model is a nested loop optimization whose computational cost rising from evaluation of the objective function and the component reliability is considerable. To improve its computational efficiency, a hybrid perturbation random moment method (HPRMM) to estimate the objective function and a hybrid perturbation inverse mapping method (HPIMM) to evaluate the component reliability will be proposed. Based on HPRMM and HPIMM, the nested loop optimization is converted into an efficient single-loop process. Effectiveness and efficiency of proposed approaches are checked by solving optimization problems of a structural–acoustic system and a disc brake system.

1. Introduction

Non-deterministic optimization methods for solving engineering problems with uncertainty factors have been receiving widespread attention [1]. Reliability-based design optimization (RBDO) is still considered the most powerful non-deterministic method [2]. In RBDO, performance and reliability of uncertain systems can simultaneously be considered. Uncertain models of RBDO can be probabilistic or non-probabilistic. In the former case, uncertain parameters are treated as probabilistic variables whose precise probability distributions are available from sampling data. This approach was successfully applied in various engineering problems. Papadrakakis and Lagaros [3] employed the neural networks (NN) and the Monte Carlo method to deal with RBDO of large-scale systems. Youn and Choi [4] developed a new RBDO methodology for large-scale structural optimization by combining the hybrid mean-value method and response surface approximation. Kaymaz and Marti [5] investigated the necessary optimality condition of the β-point for RBDO of elastoplastic mechanical structures. Lee et al. [6] proposed a FORM-based and DRM-based performance measure approach for RBDO of uncertain structures. Patel and Choi [7] employed the classification probabilistic neural networks (PNN) to accurately estimate probabilistic constraints in reliability-based topology optimization. Bichon et al. [8] constructed an efficient global surrogate model for RBDO of uncertain structures. Chen et al. [9] developed an adaptive decoupling approach based on the new update angle strategy and the novel feasibility-checking method for RBDO of uncertain structures.

In the early stage of design, the sample data to determine the precise probability distributions of uncertainties is not always sufficient. In these cases, the non-probabilistic model, such as the interval model [10], is becoming increasingly popular in RBDO. RBDO under the interval model has been successfully used in many real-life engineering problems. Chen and Wu [11] investigated RBDO of dynamic structures by using the interval extension of function and the perturbation theory of dynamic response. Luo et al. [12] solved continuum topology optimization problems with convex variables by using the adjoint variable scheme and the gradient-based algorithm. Kang et al. [13,14] investigated the non-probabilistic reliability-based topology optimization of structures by employing the performance measure-based approach and the gradient-based mathematical programming method. Chakraborty and Roy [15] employed the first order matrix perturbation theory to transform the interval optimization problem into a deterministic problem. Du [16] investigated RBDO with dependent interval variables by using the sequential optimization strategy.

For engineering problems including uncertainties with or without sufficient information to construct the corresponding precise probability distributions, a hybrid probabilistic and interval model has been developed [17]. The hybrid probabilistic and interval model was firstly used for reliability analysis of hybrid uncertain systems. Qiu et al. [18] calculated failure probability intervals of

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series systems and parallel systems by using the probability reliability theory and the interval analysis theory. Wang and Qiu [19] introduced the interval reliability model and the probabilistic operation for reliability analysis of hybrid structural systems. Guo and Du [20] developed a unified reliability analysis framework to deal with random and interval variables in multidisciplinary systems. Cao et al. [21] proposed a mixed perturbation Monte Carlo method to calculate the lower and upper bounds of structural reliabilities. Subsequently, the hybrid probabilistic and interval model has been used to analyze the response of hybrid uncertain systems. Muscolino and Sofi investigated the stochastic response of interval structures subjected to the zero-mean Gaussian random excitation [22] or to the stationary multi-correlated Gaussian random excitation [23]. Gao et al. [24] developed a hybrid probabilistic and interval method for the response analysis of structures with random and interval variables. Xia et al. proposed a probabilistic interval perturbation method for the response analysis of hybrid uncertain structural–acoustic systems [25] and hybrid uncertain acoustic fields [26]. Compared with reliability and response analyses of hybrid uncertain systems, the research on RBDO of hybrid uncertain systems is relatively lagged. RBDO of the hybrid uncertain system is a three-loop optimization problem. The inner-loop is the evaluation of extreme cases of the objective function and the limit-state function. The middle-loop is the probabilistic statistical analysis of extreme cases of the objective function and the limit-state function. The outer-loop is the optimization procedure. The computational burden rising from the implementation of the inner-loop and the middle-loop is considerable. To improve the computational efficiency of RBDO under the hybrid probabilistic and interval model, a sequential approximate programming technique was introduced into the conventional RBDO [27–29]. The nested loop optimization procedure of RBDO under the hybrid probabilistic and interval model was simplified to a series of single-loop procedures. However, if the number of single-loop procedures is large, the computational cost of the sequential approximate programming technique is still expensive. Recently, Kang et al. [30] proposed an iteration scheme based on the linear approximations of the limit-state function and the optimality condition. Based on this linearization-based approach, the nested loop problem can be converted into an approximate single-loop problem. It should be noted that the suggested linearization-based approach relies on the assumption of local monotonicity for the performance function. If the performance function is non-convex or not smooth, the global optimality cannot be guaranteed.

It appears that research on RBDO under the hybrid probabilistic and interval model is still in its preliminary stage and some important problems still remain unsolved. First, the hybrid RBDO method which converts the nested loop optimization problem into an approximate single-loop process is based on the assumption of local monotonicity for the performance function. In case of the non-convex performance function, it is still a challenge to convert the nested loop optimization problem into an approximate single-loop process. Second, hybrid RBDO methods focus on evaluating component reliability but neglect cost function uncertainty. However, since objective functions of real engineering problems may include both probabilistic and interval variables, how to deal with uncertain objective functions should also be discussed. Third, applications of hybrid RBDO methods to structural–acoustic systems and vehicle disc brake systems including both probabilistic and interval variables still are unexplored.

In view of this, the article will describe a novel hybrid RBDO method to optimize structural–acoustic systems including both probabilistic and interval variables. The present approach combines the hybrid perturbation random moment method (HPRMM) to estimate the objective function and the hybrid perturbation inverse mapping method (HPIMM) to evaluate component reliability. In HPRMM, the extreme value of objective function is calculated with the interval perturbation method, and then the stochastic perturbation method and the random moment technique are introduced to evaluate the expectation of the extreme value of the objective function. In HPIMM, the extreme value of the limit-state function is calculated with the interval perturbation method, and then the stochastic perturbation method and the inverse mapping technique serve to evaluate reliability for the extreme value of limit state function. The nested loop RBDO under the hybrid probabilistic and interval model is converted into an approximate single loop process which can be solved efficiently. The proposed RBDO method based on HPRMM and HPIMM is tested in the optimization of a structural–acoustic system and a disc brake system with a nonlinear reliability constraint.

2. RBDO under hybrid probabilistic and interval model

The reliability-based design optimization model of the structural system with both probabilistic and interval variables can be generally expressed as

$$\begin{aligned} \min_{\mathbf{d}} \quad & u(\mathbf{d},\mathbf{x},\mathbf{y}) \\ \text{s.t.} \quad & P(g_r(\mathbf{d},\mathbf{x},\mathbf{y}) \leqslant 0) \geqslant \eta_r,\ r = 1,2,\ldots,R \\ & \mathbf{d}_{\text{lower}} \leqslant \mathbf{d} \leqslant \mathbf{d}_{\text{upper}} \end{aligned} \tag{1}$$

where $\mathbf{d} = \{d_1, d_2, \ldots, d_i, \ldots\}^T$ ($i = 1, 2, \ldots, I$) is the design variable vector, and I is the number of design variables. $\mathbf{x} = \{x_1, x_2, \ldots, x_j, \ldots\}^T$ ($j = 1, 2, \ldots, J$) is the probabilistic variable vector, and J is the number of probabilistic variables. $\mathbf{y} = \{y_1, y_2, \ldots, y_l, \ldots\}^T$ ($l = 1, 2, \ldots, L$) is the interval variable vector, and L is the number of interval variables. $u(\mathbf{d},\mathbf{x},\mathbf{y})$ is the probabilistic interval objective function. $g_r(\mathbf{d},\mathbf{x},\mathbf{y})$ ($r = 1, 2, \ldots, R$) is the r-th limit-state function, and R is the number of limit-state functions. $g_r(\mathbf{d},\mathbf{x},\mathbf{y}) = 0$ is the limit-state equation by which the $\mathbf{x}$-space is divided into the safe region $g_r(\mathbf{d},\mathbf{x},\mathbf{y}) < 0$ and the failure region $g_r(\mathbf{d},\mathbf{x},\mathbf{y}) > 0$. $P(g_r(\mathbf{d},\mathbf{x},\mathbf{y}) \leqslant 0)$ is the probability for $g_r(\mathbf{d},\mathbf{x},\mathbf{y}) \leqslant 0$. η_r is the r-th design reliability index which is close to 1 with the increase of the required reliability level. $P(g_r(\mathbf{d},\mathbf{x},\mathbf{y}) \leqslant 0) \geqslant \eta_r$ is the reliability constraint.

The objective function $u(\mathbf{d},\mathbf{x},\mathbf{y})$ is a function of probabilistic and interval variables. Rising from the effect of the interval variable vector $\mathbf{y}$, $u(\mathbf{d},\mathbf{x},\mathbf{y})$ is uncertain. It is impossible and unnecessary to consider all values of $u(\mathbf{d},\mathbf{x},\mathbf{y})$. For example, in the case of structural–acoustic systems, the goal of optimization is to reduce the sound pressure in the acoustic cavity. Consequently, if the maximum value of the sound pressure response satisfies the design requirements, the other values (which are smaller than the maximum value) of the sound pressure response will also satisfy the design requirements. As a result, the maximum value of $u(\mathbf{d},\mathbf{x},\mathbf{y})$ is selected as the optimization objective. The maximum value of the objective function $u(\mathbf{d},\mathbf{x},\mathbf{y})$ can be obtained by solving the following sub-optimization problem

$$\begin{aligned} \max \quad & u(\mathbf{d},\mathbf{x},\mathbf{y}) \\ \text{s.t.} \quad & \underline{\mathbf{y}} \leqslant \mathbf{y} \leqslant \bar{\mathbf{y}} \end{aligned} \tag{2}$$

where $\underline{\mathbf{y}}$ and $\bar{\mathbf{y}}$ are the lower and upper bounds of the interval variable vector $\mathbf{y}$.

Due to the effects of probabilistic variables, the maximum value of $u(\mathbf{d},\mathbf{x},\mathbf{y})$ is also uncertain. The probabilistic statistic characteristic of the maximum value of $u(\mathbf{d},\mathbf{x},\mathbf{y})$ can be expressed as the expectation of the maximum value of $u(\mathbf{d},\mathbf{x},\mathbf{y})$. Therefore, the purpose of RBDO under the hybrid probabilistic and interval model can be expressed as

$$\min_{\mathbf{d}} \quad \mu(\bar{u}(\mathbf{d},\mathbf{x},\mathbf{y})) \tag{3}$$