Journal of Econometrics 167 (2012) 458–472

Contents lists available at SciVerse ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Semiparametric inference in a GARCH-in-mean model

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ARTICLE INFO

Article history:
Available online 15 October 2011

JEL classification:
C13
C14
C22
G12

Keywords:
Efficiency bound
GARCH-M model
Profile likelihood
Risk-return relation
Semiparametric inference

ABSTRACT

A new semiparametric estimator for an empirical asset pricing model with general nonparametric risk-return tradeoff and GARCH-type underlying volatility is introduced. Based on the profile likelihood approach, it does not rely on any initial parametric estimator of the conditional mean function, and it is under stated conditions consistent, asymptotically normal, and efficient, i.e., it achieves the semiparametric lower bound. A sampling experiment provides finite sample comparisons with the parametric approach and the iterative semiparametric approach with parametric initial estimate of Conrad and Mammen (2008). An application to daily stock market returns suggests that the risk-return relation is indeed nonlinear.

1. Introduction

The relation between risk and return is of central importance in asset pricing, hedging, derivative pricing, and risk management. The intertemporal capital asset pricing model (ICAPM) of Merton (1973) predicts a positive and linear relation between the expectation and the variance of returns. Essentially, investors must be compensated for bearing additional risk. Perhaps surprisingly, both significance and even the sign of the linear relation between expected return and variance of return have proved elusive in empirical work.

In the present paper, we explore the possibility that the mixed empirical evidence may be due to misspecification of the functional form of the risk-return relation. We allow for a general nonparametric risk-return tradeoff, and model the conditional variances as a GARCH-type process. Besides the possibly nonlinear GARCH-in-mean effect, our specification accommodates exogenous regressors that are typically used as conditioning variables entering linearly in the mean equation, such as the dividend yield. We introduce a new semiparametric estimation procedure for the resulting model that does not rely on an initial parametric (linear) estimate of the risk-return relation, which would necessarily be inconsistent if the true relation is indeed nonlinear, and this feature is the key to establishing the asymptotic properties of our estimator. Using the profile likelihood approach, we prove that our semiparametric estimator is consistent, asymptotically normal, and achieves the semiparametric efficiency bound.

The literature on the risk-return tradeoff is massive. Motivated by the ICAPM, the original ARCH-M model proposed by Engle et al. (1987) introduces conditional variance into the conditional mean return equation in a linear fashion. Empirical studies of the risk-return tradeoff applying GARCH-type models to stock returns have obtained mixed results on both the sign and significance of the in-mean effect, see, e.g., Bollerslev et al. (1988), Chou (1988), Nelson (1991), Campbell and Hentschel (1992), Chou et al. (1992), Backus and Gregory (1993), Glosten et al. (1993), and Harrison and Zhang (1999). Poterba and Summers (1986) show that the stock market level is determined by the risk-return tradeoff in conjunction with the degree of serial correlation in volatility. Indeed, recent work in asset pricing focusing on volatility innovations examines cross-sectional risk premia induced by covariance between volatility changes and stock returns and finds negative premia, e.g., Ang et al. (2006). The idea is that since innovations in volatility are higher during recessions, stocks that co-vary with volatility pay off in bad states, and so should require smaller risk premia. Christensen et al. (2010) consider aggregate time series data on returns and

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doi:10.1016/j.jeconom.2011.09.028

innovations in GARCH-volatilities and confirm the negative premia from the cross-sectional studies. On the other hand, a positive risk-return tradeoff has been indicated by Brandt and Kang (2004) using a latent VAR methodology, by Ghysels et al. (2005) using weighted rolling sample windows in the variance measurements, and by Christensen and Nielsen (2007) who consider innovations in realized as well as option-implied volatility. For a survey of related studies, see Lettau and Ludvigson (2010).

One possible source of the mixed results is misspecification of the way in which conditional variance enters the conditional mean return equation. Indeed, in his early empirical study, Merton (1980) regressed returns not only on sample variances, but also on realized sample standard deviations (volatilities) of returns over subintervals, to determine which relation was more stable. The coefficient in the first regression would be interpreted as the Arrow–Pratt rate of relative risk aversion of the representative investor in the ICAPM, and that in the alternative (squareroot) version of the regression as the slope of the capital market line or Sharpe ratio in the static CAPM of Sharpe (1964) and Lintner (1965). Considerations and debates of these kinds have lead to the interest in specifying more flexible models of the risk-return relation. Flexibility, however, comes at the cost of more complicated statistical properties. Linton and Perron (2003) use a mean equation given by

$$y_t = \mu\left(\sigma_t^2\right) + \varepsilon_t\sigma_t, \tag{1}$$

where y_t is the daily return, σ_t^2 is the conditional return variance given information available up to time $t-1$, $\varepsilon_t \sim$ i.i.d. (0, 1), and $\mu(\cdot)$ is a smooth mean function determining the functional form of the risk-return relation. The specification is estimated semiparametrically, using an EGARCH process for the conditional variance, but no asymptotic theory is provided. As the authors state, writing the smooth mean function given the other parameters ϕ as $\mu_\phi(\cdot)$, "$\cdots$unfortunately, in our model, we cannot define the corresponding profile quantity $\widehat{\mu}_\phi\left(\sigma_t^2\right)$ so easily, since σ_t^2 depends, in addition to the parameters, on lagged $\varepsilon's$, which in turn depend on lagged $\mu's$. Therefore, we need to know the entire function $\mu(\cdot)$ (or at least its values at the T sample points) to construct $\widehat{\mu}_\phi\left(\sigma_t^2\right)$". Conrad and Mammen (2008) propose an algorithm and a specification test for GARCH-M effects of the type (1), using QML to get starting values, but they actually require a consistent estimator for the starting values (e.g., in their Assumption 5), and the QML estimator they use is necessarily inconsistent if $\mu(\cdot)$ is indeed nonlinear. Further, their main tool is empirical process theory, and this involves high level assumptions, such as $E\left[\exp\left(\rho\left|\varepsilon_t\right|\right)\right] < \infty$ for $\rho > 0$. Hodgson and Vorkink (2003) estimate the density function of a multivariate GARCH-M model in a semiparametric fashion, but also do not provide a formal asymptotic theory. Sun and Stengos (2006) propose yet another type of semiparametric GARCH-M model, but the nonparametric portion is the density of the innovations, whereas the conditional means and standard deviations are proportional (we reject this case empirically, using a general $\mu(\cdot)$).

In this paper, we consider an alternative approach based on (1) and establish the asymptotic theory necessary for inference. The two main differences between our approach and that of Conrad and Mammen (2008) are that (i) we do not use QML or any other inconsistent estimator as starting value, and (ii) instead of empirical process theory we use a profile likelihood approach. In Section 3.4 we also extend our model to allow for exogenous covariates in the conditional mean equation. We could have added covariates in the conditional variance equation, following Campbell (1993), who found a negative risk-return relation in a parametric model in this case. However, Han and Park (2008) and Iglesias (2009) show that allowing for covariates in the conditional variance equation leads to difficulties, and we leave the extension for future research. The model of the present paper is an extension of the double autoregressive model of Ling (2004) to include a general risk-return relation, and is amenable to asymptotic analysis based on the profile likelihood methodology, along the lines of Severini and Wong (1992). Ling (2004) provides empirical support favoring the double autoregressive model over the traditional autoregressive-ARCH in a number of financial return series. Dahl and Iglesias (2011) provide evidence of further cases where volatility is driven by functionals of data, as in the double autoregressive case. In the present paper, we establish the empirical relevance of introducing a risk premium in the double autoregressive model.

Our estimation procedure is easy to apply and readily allows calculation of consistent standard errors. In contrast to alternatives such as adaptive estimation, the profile likelihood approach does not require the matrix of expected second order derivatives with respect to the parameters of interest and the nuisance parameters to be block diagonal. Indeed, block diagonality is violated in our model, as we show. The profile approach is based on the principle that a semiparametric problem is at least "as hard" as any parametric subproblem. Therefore, the Fisher information for estimating the parameter of interest in a semiparametric problem is not greater than the Fisher information for estimating that parameter in any parametric subproblem. Hence, we may look at the "least favorable subproblem" and obtain a lower bound on the asymptotic variance of the parameter of interest in the original semiparametric problem. In our case, the parameters of the underlying GARCH process play the role as the parameters of interest, and in this way they become robust to possible nonlinearity of unspecified form in the conditional mean function.

Our asymptotic theory utilizes the classic Cramér type conditions for consistency and asymptotic normality, i.e., a central limit theorem for the score, convergence of the Hessian, and uniformly bounded third order derivatives (see, e.g., Lehmann (1999) and Jensen and Rahbek (2004a,b)). This third order approach works through local identification, rather than assuming identification at the outset, and hence we do not require an initial consistent estimate of the conditional mean function. We demonstrate which μ-functions are permitted under this approach (we provide sufficient conditions). Building up the analysis in steps, we first establish the asymptotic theory for the case of a known "curve" defining the relevant subproblem, e.g., $\mu\left(\sigma_t^2\right)$ may be given by $\lambda(\phi)\sigma_t^2$ or $\lambda(\phi)\sigma_t$, for a known function $\lambda(\phi)$ giving the relative risk aversion respectively Sharpe ratio corresponding to given values of the GARCH parameters ϕ. This part of the paper provides the first asymptotic theory for parametric GARCH-M models. Based on this, we then go on to the general semiparametric case of unknown curves and provide the required consistent estimator of a least favorable curve (or subproblem).

The paper is laid out as follows. Section 2 describes our general strategy, based on the profile likelihood approach and the estimation of a least favorable curve. The presentation is heuristic, intended to provide intuition, and leaving technical details to later sections. Section 3 presents our new model and semiparametric estimator. We state conditions under which our estimator is consistent, asymptotically normal, and attains the semiparametric lower bound, without relying on an initial consistent estimate. Section 4 describes in detail our semiparametric estimation algorithm. In a sampling experiment we explore finite sample accuracy, comparing with the parametric approach and the iterative semiparametric approach with parametric initial estimate proposed by Conrad and Mammen (2008). Finally, an empirical application to daily stock market returns is offered. Section 5 concludes. Appendix A collects the proofs of lemmas and theorems. Appendix B contains information about from where to obtain supplementary material related to this article.