Journal of Econometrics 167 (2012) 473–482

Contents lists available at SciVerse ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

A semiparametric stochastic volatility model[☆]

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ARTICLE INFO

Article history:
Available online 1 October 2011

JEL classification:
G12
C11
C15

Keywords:
Leverage effect
Simulated maximum likelihood
Laplace approximation
Spline

ABSTRACT

In this paper the correlation structure in the classical leverage stochastic volatility (SV) model is generalized based on a linear spline. In the new model the correlation between the return and volatility innovations is time varying and depends nonparametrically on the type of news arrived to the market. Theoretical properties of the proposed model are examined. The model estimation and comparison are conducted by Bayesian methods. The performance of the estimates are examined in simulations. The new model is fitted to daily and weekly US data and compared with the classical SV and GARCH models in terms of their in-sample and out-of-sample performances. Empirical results suggest evidence in favor of the proposed model. In particular, the new model finds strong evidence of time varying leverage effect in individual stocks when the classical model fails to identify the leverage effect.

1. Introduction

How volatility responds to return news has long been an active research topic; see Black (1976), Christie (1982), Engle and Ng (1993) and Wu and Xiao (2002) for a rather incomplete list of studies in the literature. Answer to this question has important implications for financial decision making and asset pricing. For example, predictability of volatility critically depends on the relationship between the return shock and volatility. Moreover, there are important implications of the relationship for portfolio selection and risk management (Bekaert and Wu, 2000) and for "betas" (Braun et al., 1995). Furthermore, an option contract would be substantially mis-priced when the relationship is misspecified (Duan, 1995).

It is now well accepted in the volatility literature that equity volatility responds asymmetrically to return news, namely, a piece of bad news has different impact on future volatility from the good news of the same magnitude. The most popular and convenient empirical method for examining the asymmetric volatility response is via some form of ARCH-type models. The motivation mainly comes from the so-called leverage hypothesis originally put forward by Black (1976). According to the leverage hypothesis, when bad news arrives, it decreases the value of a firm's equity and hence increases its leverage. Consequently, the equity becomes more risky and its volatility increases. Likewise the volatility decreases after good news arrives.

Volatility response can also be studied using stochastic volatility (SV) models. Unlike ARCH-type models, SV models specify volatility as a separate random process, which provides certain advantages over the ARCH-type models for modeling the dynamics of asset returns (Kim et al., 1998). The third method for studying volatility response is to use realized volatility; see, for example, Andersen et al. (2001, ABDE hereafter), Bandi and Reno (forthcoming) and Hansen et al. (2010). In this literature some important asymmetries are well documented in market-wide equity index returns but not in individual stocks. This observation leads some researchers to conclude that the significant asymmetries in equity index returns are due to volatility feedback effect but not leverage effect; see ABDE.

In the SV literature, the asymmetric volatility response is often studied by specifying a negative correlation between the return innovation and the volatility innovation. This classical leverage SV model was first estimated by Harvey and Shephard (1996). The model specification requires the correlation coefficient between the two innovations remains constant, regardless of price movements. On the other hand, Daouk and Ng (2007) reported evidence of stronger leverage effect in down markets than in up markets. Obviously, this empirical result cannot be explained by the classical leverage SV model with a constant leverage effect.

The central focus of the present paper is to provide a more general framework to investigate the asymmetric relationship

☆ I wish to thank the co-editors, referees, Yong Li, Peter Phillips, Neil Shephard, Liangjun Su, Anthony Tay, Yiu Kuen Tse, seminar participants at Singapore Management University, FEMES, NZESG, SESG, London School of Economics, FEMES, SETA2008, and CREST for comments on an earlier version of the paper. Excellent research assistantship from Chao Luo and Xin Chen are acknowledged. I gratefully acknowledge financial support from the Singapore Ministry of Education AcRF Tier 2 fund under Grant No. T206B4301-RS.

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doi:10.1016/j.jeconom.2011.09.029

between volatility and return news in the context of SV models. Using the linear spline, we allow the correlation coefficient between the two innovations to be time varying and depend nonparametrically on the size and the direction of the previous price movement. Since our model nests the SV model with the constant leverage, we can easily check the validity of this classical specification. Empirical applications reveal strong evidence against the classical specification both in-the-sample and out-of-the-sample.

Our model extends the specification studied in Harvey and Shephard (1996), Yu (2005) and Omori et al. (2007). Following Meyer and Yu (2000), the Bayesian Markov chain Monte Carlo (MCMC) methods are used to estimate and compare alternative models. Our model is closely related to the model of Wu and Xiao (2002) where a flexible nonparametric model was used to relate the log implied volatility and the lagged return innovation. However, our work is different from Wu and Xiao in four aspects. First, Wu and Xiao is an ARCH-type model while ours is an SV. The two models do not nest each other. Although the model of Wu and Xiao allows for a very general news impact function, it assumes an additive functional form and cannot even nest the simplest SV model. Second, different nonparametric methods are employed. While we use the spline-based smoother, Wu and Xiao used the Nadaraya–Watson kernel method in a partial linear framework. One of the main advantages for the kernel method lies in its simpler theoretical analysis. However, the kernel method cannot be used in the context of SV due to the curse-of-dimensionality problem. Third, the relationship between return and log-volatility is in the physical measure in our study but is in the risk-neutral measure in theirs. The risk-neutral measure is more useful for pricing whereas the physical measure allows one to forecast volatility. Finally, volatility is latent in our method whereas Wu and Xiao assumed that the volatility of the US market index is well approximated by the volatility index, VIX. For individual stocks, VIX is no longer a valid approximation to the volatility.

Our model is somewhat related to that of Engle and Ng (1993) in the sense that the linear spline is used. However, we use the linear spline to model the correlation between the two innovations while Engle and Ng used it as a regression tool to relate volatility to the lagged return innovation. Robinson (1991) and others provided more general ARCH models. All the models are of an additive structure and hence do not nest ours. Finally, our model is related to Bandi and Reno (forthcoming) where the time varying leverage effect is estimated using a nonparametric method with intra-day data. Unlike Bandi and Reno who tie the strength of the leverage effect to the current level of volatility, we assume the driving factor for the time varying leverage is the lagged return.

The article is organized as follows. In Section 2 we introduce the semiparametric SV model and develop some statistical properties of the model. Section 3 discusses the MCMC methods for parameter estimation and for model comparison and documents the performance of MCMC in simulations. Empirical results based on US data are presented and discussed in Section 4. Section 5 concludes. Appendix proves the theorem.

2. The proposed SV model

Let y_t be the rate of return of a stock or a market portfolio in time period t, σ_t^2 be the conditional variance of y_t, $h_t = \ln \sigma_t^2$, ϵ_t be the return innovation. GARCH models specify a deterministic relationship between σ_{t+1}^2 and y_t (or ϵ_t). Different models coexist to capture the asymmetric volatility response. For example, EGARCH(1, 1) of Nelson (1991) assumes

$$h_{t+1} = \alpha + \varphi h_t + \beta_0 \epsilon_t + \beta_1 |\epsilon_t|, \tag{1}$$

where the asymmetry is induced by the term $\beta_0 \epsilon_t$. Threshold GARCH(1, 1) of Glosten et al. (1993) assumes

$$\sigma_{t+1}^2 = \alpha + \varphi \sigma_t^2 + \beta y_t^2 + \beta^* y_t^2 \mathbf{1}(y_t < 0), \tag{2}$$

where $\mathbf{1}(y_t < 0) = 1$ if $y_t < 0$ and 0 otherwise. In this model, the asymmetry is induced by $\mathbf{1}(\cdot)$. However, based on a nonparametric technique, Mishra et al. (2010) have found the evidence of further asymmetry in the residuals of fitted threshold GARCH(1, 1).

Engle and Ng (1993) introduced a partially nonparametric model of the form

$$\sigma_{t+1}^2 = \alpha + \varphi \sigma_t^2 + m(\epsilon_t) \tag{3}$$

where $m(\cdot)$ is an unknown function. Engle and Ng estimated $m(\cdot)$ using the linear spline

$$m(\epsilon_t) = \sum_{i=0}^{m^+} \theta_i \mathbf{1}(\epsilon_t > \tau_i)(\epsilon_t - \tau_i) + \sum_{i=0}^{m^-} \delta_i \mathbf{1}(\epsilon_t < \tau_{-i})(\epsilon_t - \tau_{-i}),$$

where τ_i are the predetermined knots associated with the linear spline.

In contrast to ARCH-type models, the SV models specify a stochastic relationship between σ_{t+1}^2 (or h_{t+1}) and y_t by using an additional innovation. It is very important to point out that the meaning of σ_{t+1}^2 in SV models is NOT the same as that in ARCH-type models. By assuming σ_{t+1}^2 is a conditional variance, ARCH-type models adopt the one-step-ahead prediction approach to volatility modeling. Whereas, due to the presence of an additional innovation in the state equation of SV, σ_{t+1}^2 is not measurable with respect to the natural filtration and hence is not a conditional variance. This difference has an important implication for the analysis of the news impact, which will be discussed in detail later.

To account for volatility asymmetry, the classical leverage SV model takes the form of

$$y_t = \mu_y + \sigma \exp(h_t/2)\epsilon_t, \quad \epsilon_t \sim \text{i.i.d. } N(0,1), \tag{4}$$

$$h_{t+1} = \varphi h_t + \gamma v_t, \quad v_t \sim \text{i.i.d. } N(0,1), \tag{5}$$

where $\text{corr}(\epsilon_t, v_t) = \rho$. Eq. (5) can be equivalently represented by

$$h_{t+1} = \varphi h_t + \gamma(\rho \epsilon_t + \sqrt{1-\rho^2} w_t), \tag{6}$$

where w_t is i.i.d. $N(0, 1)$ and $\text{corr}(\epsilon_t, w_t) = 0$. Consequently, we have

$$\begin{aligned} h_{t+1} &= \varphi h_t + \gamma \rho \epsilon_t + \gamma \sqrt{1-\rho^2} w_t \\ &= \varphi h_t + \rho \frac{\gamma}{\sigma} \exp(-h_t/2)(y_t - \mu_y) + \gamma \sqrt{1-\rho^2} w_t, \end{aligned} \tag{7}$$

implying that on average $\ln \sigma_{t+1}^2$ is a linear function in y_t. When $\rho < 0$, the linear function is downward sloping and this feature is often referred to as the leverage effect. Clearly the relationship between $\ln \sigma_{t+1}^2$ and y_t is independent of the sign and the size of ϵ_t and hence the leverage effect, captured by ρ, is a constant in this model.

There is ample evidence that the effect of bad news on volatility is different from the good news of the same magnitude. Using the firm level accounting data, Figlewski and Wang (2000) reported a more remarkable leverage effect in down markets than in up markets. A similar pattern of asymmetry found in Daouk and Ng (2007) using unleveled firm volatility. The evident suggests that a global linear relationship between $\ln \sigma_{t+1}^2$ and y_t may be too restrictive and there is a clear need for a more general SV model for the volatility asymmetry.

To introduce our semiparametric SV model, we first choose m knots, denoted by $\tau_1, \ldots, \tau_m$ with $\tau_1 > \cdots > \tau_m$, from the support