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Estimation of fractional integration under temporal aggregation

Uwe Hassler *

Goethe University Frankfurt, RuW, Grueneburgplatz 1, 60323 Frankfurt, Germany

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ABSTRACT

A result characterizing the effect of temporal aggregation in the frequency domain is known for arbitrary stationary processes and generalized for difference-stationary processes here. Temporal aggregation includes cumulation of flow variables as well as systematic (or skip) sampling of stock variables. Next, the aggregation result is applied to fractionally integrated processes. In particular, it is investigated whether typical frequency domain assumptions made for semiparametric estimation and inference are closed with respect to aggregation. With these findings it is spelled out, which estimators remain valid upon aggregation under which conditions on bandwidth selection.

1. Introduction

Determining inflation persistence is a prominent issue when it comes to forecasting (Stock and Watson, 2007), or when monetary policy recommendations are at stake; see e.g. Mishkin (2007). The effect of temporal aggregation on inflation persistence has recently been studied by Paya et al. (2007). Fractional integration is one model for inflation persistence that can be traced back to Hassler and Wolters (1995) or Baillie et al. (1996). The question how aggregation and persistence interact is of interest beyond inflation, and has troubled applied economists for a long time; see Christiano et al. (1991) for empirical evidence in the context of the permanent income hypothesis and Rossana and Seater (1995) for a representative set of economic time series. Using fractionally integrated models, Chambers (1998) found with macroeconomic series that the empirical degree of integration may depend on the level of temporal aggregation; see also Diebold and Rudebusch (1989) or Tschernig (1995). In empirical finance, too, one of the core issues with respect to realized volatility is optimal sampling; see e.g. Ait-Sahalia et al. (2005) and the results by Drost and Nijman (1993).

In this paper we understand by temporal aggregation both systematic sampling (or skip sampling) of stock variables where only every pth data point is observed, and summation of flow variables where neighboring observations are cumulated to determine the total flow. Econometricians have devoted their attention to both types of temporal aggregation for decades; see Silvestrini and Veredas (2008) for a recent survey. Early results for autoregressive moving average (ARMA) models were obtained by Brewer (1973) and Weiss (1984). A treatment of integrated (of order one) ARIMA models was provided by Wei (1981) and Stram and Wei (1986), for skip sampling and cumulating, respectively. In particular, skip sampling can be embedded in the more general problem of missing observations; see Palm and Nijman (1984) for an investigation of dynamic regression models. Aspects of forecasting have been addressed by Lütkepohl (1987) and Lütkepohl (2009), while Marcellino (1999) deals with cointegration and causality under aggregation. Moreover, the potential interaction of seasonal integration and unit roots at frequency zero due to temporal aggregation was studied by Granger and Siklos (1995); see also Pons (2006). In fact, there is a literature on "span versus frequency" when it comes to testing the null hypothesis of a unit root, which started with Shiller and Perron (1985) and came to a preliminary end with Chambers (2004).

Notwithstanding the vast amount of papers on temporal aggregation, little attention has been paid to effects in the frequency domain, notable exceptions being Drost (1994) and Souza (2003). In the frequency domain, temporal aggregation is accompanied by the so-called aliasing effect, which is well known under discrete-time sampling from a continuous-time process; see e.g. Hansen

* Tel.: +49 69 798 34762; fax: +49 69 798 35014.
E-mail address: hassler@wiwi.uni-frankfurt.de.

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and Sargent (1983). For the special case of fractional integration, spectral results have been obtained by Chambers (1998), Hwang (2000), Tsai and Chan (2005b), and Souza (2005). Further, Chambers (1996) and Tsai and Chan (2005a) cover the related case of discrete-time sampling from a continuous-time long memory process, while Souza (2007, 2008) focusses on the effect of temporal aggregation on widely used memory estimators.

We add two aspects to this literature: a general characterization of time aggregation in the frequency domain for processes that become stationary only after differencing r times for some natural number r, and an investigation, which semiparametric estimators of fractionally integrated models retain their consistency and limiting normality under aggregation. In greater detail our contributions are the following. We draw from the literature results on aliasing and moving-averaging in case of temporal aggregation of arbitrary stationary processes (Lemmas 1 and 2), and we combine these lemmae to characterize the frequency domain effect of temporal aggregation for processes that become stationary only after integer differencing r times, $r = 0, 1, 2, \ldots$ (Proposition 1). Next, the aggregation results are applied to fractionally integrated processes. In particular, we investigate whether typical assumptions on fractionally integrated processes, which are made in the literature to obtain consistency or limiting normality of semiparametric estimators, are closed with respect to aggregation. In other words: if $\{z_t\}$ satisfies a set of assumptions used to prove properties of some estimator or test, does the temporal aggregate fulfill them, too? Differing findings are obtained for cumulating of flow data (Proposition 2), skip sampling of stocks (Proposition 3), and for the case of generalized fractional integration where the singularity may occur at frequencies different from zero (Proposition 4). In a couple of remarks we discuss as consequences for applied work, which estimators remain valid upon aggregation (under which conditions on the bandwidth choice).

The rest of this paper is organized as follows. Section 2 treats the general aggregation effect in terms of spectral densities. In Section 3, the aggregation results are applied to the semiparametric estimation of the memory parameter of fractional integration. The last section contains a more detailed non-technical summary. Proofs are relegated to the Appendix.

2. Aggregation in the frequency domain

For sequences $\{a_j\}$ and $\{b_j\}$, let $a_j \sim b_j$ denote $a_j/b_j \to 1$ as $j \to \infty$, while for functions, $a(x) \sim b(x)$ is short for $a(x)/b(x) \to 1$ as $x \to 0$. Further, $a(x) = O(x^c)$ means that $a(x)x^{-c}$ is bounded as $x \to 0$, while $a(x) = o(x^c)$ signifies $a(x)x^{-c} \to 0$. First-order derivatives are given as $a'(x)$. Finally, let $\mathbb{Z}$ stand for the set of all integers.

2.1. Notation and assumptions

Let $\{z_t\}$, $t = 1, 2, \ldots, T$, denote some time series to be aggregated over p periods. The aggregate is constructed for the new time scale τ. In case of stock variables, aggregation or systematic sampling means *skip sampling* where only every p'th data point is observed,

$$\dot{z}_\tau := z_{p\tau}, \quad \tau = 1, 2, \ldots, \tag{1}$$

where for the rest of the paper $p \geq 2$ is a finite integer. Flow variables are aggregated by *cumulating* p neighboring observations that do not overlap to determine the total flow over p sub-periods,

$$\begin{aligned}\widetilde{z}_\tau &:= z_{p\tau} + z_{p\tau-1} + \cdots + z_{p(\tau-1)+1} \\ &= S_p(L) z_{p\tau}, \quad \tau = 1, 2, \ldots,\end{aligned} \tag{2}$$

where $S_p(L) := 1 + L + \cdots + L^{p-1}$ is the moving average polynomial of degree p in the usual lag operator L. Hence, $\{\widetilde{z}_\tau\}$ is obtained by skip sampling the overlapping moving average process $\{S_p(L)z_t\}$.

Clearly, many economic variables are not stationary. It is often assumed that the basic variable $\{z_t\}$ is given by integration over stationary increments,

$$z_t = z_0 + \sum_{i=1}^{t} y_i, \quad t = 1, 2, \ldots, T.$$

If $\{y_t\}$ is a stationary fractionally integrated process of order d, $d < 0.5$, as defined in a subsequent section, then the partial sum process $\{z_t\}$ is sometimes called fractionally integrated (of order $\delta = 1 + d$) of "type I"; see Marinucci and Robinson (1999) and Robinson (2005). Some economic variables are even considered as integrated of order 2. Therefore, we allow for stationarity and different degrees of nonstationarity at the same time. It is maintained for some natural number $r \in \{0, 1, 2, \ldots\}$ that the process $\{z_t\}$ solves the following difference equation with $\Delta = 1 - L$:

$$\Delta^r z_t = y_t, \quad t = 1, 2, \ldots, T. \tag{3}$$

Note that differencing changes the status of stock series: While log-prices $p_t = \log P_t$ are stocks, the inflation rate $\pi_t = \Delta p_t$ is a flow variable.

To fully specify the potentially nonstationary processes from (3), we have to add assumptions on $\{y_t\}$. Our results will hold for any stationary process $\{y_t\}$ with integrable spectral density f_y. Since f_y is an even and 2π-periodic function, the definition of the spectral density can be extended to the whole real range, and we focus on the interval $[0, \pi]$ in the following assumption.

Assumption 1. The process $\{y_t\}$, $t \in \mathbb{Z}$, is covariance stationary with integrable spectral density $f_y(\lambda)$ on Π, where $\Pi = [0, \pi]$ if f_y is well defined on the whole interval, or $\Pi = [0, \pi] \setminus \{\lambda^*\}$ if f_y has a singularity at some frequency $\lambda^* \in [0, \pi]$.

Note that f_y does not have to exist everywhere. A singularity at λ^* might come from (generalized) fractional integration with long memory; see (12) below. In fact, we might allow for k singularities (having e.g. so-called k-factor Gegenbauer processes in mind; see Woodward et al., 1998). Further, we stress that $f_y(0) = 0$ is not excluded. This covers the particular case of over-differencing. Assume e.g. that no differencing is required to obtain stationarity, but $\{z_t\}$ is differenced in practice. This case is dealt with by $r = 1$ in (3) with the assumption that $\{y_t\}$ is over-differenced.

To set the scene for the next subsection, we define the lag operator $\mathcal{L}$ operating on the aggregate time scale τ, such that $\mathcal{L} = L^p$ with L operating on t (see e.g. Wei, 1990, Ch. 16). Let $\nabla = 1 - \mathcal{L}$ stand for the differences of the new time scale τ. In case that $r \geq 1$ in (3), we will study the effect of first aggregating and then differencing. The spectral densities of the differenced aggregates $\{\nabla^r \dot{z}_\tau\}$ and $\{\nabla^r \widetilde{z}_\tau\}$ are denoted as $\dot{f}_{\nabla^r z}(\lambda)$ and $\widetilde{f}_{\nabla^r z}(\lambda)$, respectively. For $r = 0$, we have $z_t = y_t$ and $\dot{f}_y(\lambda)$ or $\widetilde{f}_y(\lambda)$ represent the spectra of the stationary aggregates $\{\dot{y}_\tau\}$ and $\{\widetilde{y}_\tau\}$.[1]

2.2. Result and discussion

The main effect in the frequency domain is the so-called aliasing effect that arises from skip sampling. Since cumulation of non-overlapping data can be reduced to skip sampling a moving average, the effect will be present also with flow data. Therefore, we first pin down the aliasing effect. The following finding for

[1] Sometimes stock variables are aggregated by averaging over p non-overlapping observations, $\{\bar{z}_\tau\}$, such that p sub-periods are replaced by the mean of p values. Obviously this is directly connected to cumulation from (2), $\bar{z}_\tau := \widetilde{z}_\tau/p$. Let the spectrum of the differenced aggregate $\{\nabla^r \bar{z}_\tau\}$ be denoted as $\bar{f}_{\nabla^r z}(\lambda)$. There is no need to address the case of averaging separately since it holds $\bar{f}_{\nabla^r z}(\lambda) = \widetilde{f}_{\nabla^r z}(\lambda)/p^2$.