Computers and Structures 157 (2015) 31–41

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Topology optimization of reinforced concrete structures considering control of shrinkage and strength failure

Yangjun Luo [a,b,*], Michael Yu Wang [c], Mingdong Zhou [d], Zichen Deng [b]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China
[b] School of Mechanics, Civil Engineering & Architecture, Northwestern Polytechnical University, Xi'an 710072, China
[c] Mechanical Engineering Department, National University of Singapore, EA-05-11, 9 Engineering Drive 1, Singapore
[d] Department of Mechanical Engineering, Technical University of Denmark, DK-2800 Kgs Lyngby, Denmark

ARTICLE INFO

Article history:
Received 17 January 2015
Accepted 5 May 2015
Available online 28 May 2015

Keywords:
Reinforced concrete
Shrinkage
Topology optimization
Drucker–Prager criterion
Enhanced aggregation method

ABSTRACT

To take into account the shrinkage effect in the early stage of Reinforced Concrete (RC) design, an effective continuum topology optimization method is presented in this paper. Based on the power-law interpolation, shrinkage of concrete is numerically simulated by introducing an additional design-dependent force. Under multi-axial stress conditions, the concrete failure surface is well fitted by two Drucker–Prager yield functions. The optimization problem aims at minimizing the cost function under yield strength constraints on concrete elements and a structural shrinkage volume constraint. In conjunction with the adjoint-variable sensitivity information, the enhanced aggregation method is utilized to efficiently reduce the computational effort arisen from large-scale strength constraints. Numerical results reveal that the proposed approach can produce a reasonable solution with the least steel reinforcements to ensure the structural safety under the combined action of external loads and shrinkage.

1. Introduction

Unlike steel of substantial capability in withstanding both tensile and compressive stresses, concrete is safely designed only for compression, which makes plain concrete unacceptable for most engineering structures. To expand its application range, concrete is commonly reinforced with steel bars at regions where internal tension occurs, *i.e.* Reinforced Concrete (RC). Among various techniques for the layout planning of steel reinforcements within concrete, a widely acknowledged method is the so-called "strut-and-tie model" (STM) [1], which has been recommended by many code provisions, *e.g.* Eurocode 2 [2] and ACI building code [3]. In the STM method, the continuum RC structure is reduced to an equivalent truss-like structure using the stress trajectories or load path methods. For a structure with complex geometric and loading conditions, the STM method has been combined with linear programming [4,5] and the genetic algorithms [6] to improve efficiency in automatically seeking the optimal placement of reinforcements.

Continuum topology optimization provides designers with more design freedom, and it has been greatly developed and applied to various engineering fields since the pioneering work of Bendsøe and Kikuchi [7]. An overview and survey can be found in the literatures of Eschenauer and Olhoff [8], Bendsøe and Sigmund [9]. In the context of utilizing continuum topology optimization for RC design, early efforts focus on the topology optimization methods with single linear-elastic material. Typically, Kwak and Noh [10], Liang et al. [11] generated the strut-and-tie model of two-dimensional RC structures by using the evolutionary structural optimization (ESO) method. Leu et al. [12] further extended the application of ESO to three-dimensional RC members. Bruggi [13] resorted the minimum compliance topology optimization to determine the preliminary truss-like patterns for STM. Their results were very similar to those obtained with the pure steel structure using classical topology optimization. Aiming at incorporating different mechanical properties of steel and concrete into the topology optimization, Victoria et al. [14] presented an optimality criterion method for the T&C design of strut-and-tie models. More recently, the tension–compression asymmetry property of concrete has received attention in the RC layout design. Luo and Kang [15] suggested a two-material topology optimization with Drucker–Prager stress constraints for un-cracked RC structures. Bogomolny and Amir [16] implemented the design of RC structures on the basis of topology optimization with elastoplastic material

* Corresponding author at: State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China. Tel.: +86 41184706067; fax: +852 41184706202.

E-mail address: yangjunluo@dlut.edu.cn (Y. Luo).

http://dx.doi.org/10.1016/j.compstruc.2015.05.009
0045-7949/

model considering Drucker–Prager criteria and post yielding response. In addition, Bruggi and Taliercio [17] studied the topology optimization problem of fiber-reinforcement retrofitting existing structures. Gaynor et al. [18] employed a hybrid truss-continuum topology optimization to design the STM of reinforced and prestressed concrete members. Amir and Sigmund [19] proposed a continuum and truss topology optimization method, in which concrete was represented by a damage model with strain-softening. It was shown that reasonable layouts of reinforcements in RC structures can be achieved.

Aside from its relatively low tensile strength, the material properties of concrete that are most significant in terms of RC structural behavior are: (1) brittleness and (2) shrinkage. The brittleness may result in sudden, catastrophic failure under ultimate loads. To prevent this, imposing the strength failure constraint on concrete is necessary for optimal design of RC structures. Moreover, concrete shrinks when it dissipates the hydration heat as the concrete hardens (named by thermal shrinkage) and loses moisture in a drying environment (named by drying shrinkage). If such shrinkage is restrained by the action of steel reinforcements or the fixity of boundary conditions, there will be additional tensile stresses within the concrete. The magnitude of the final shrinkage strain depends on the composition of the concrete and the external environment. Typical final shrinkage strains of concrete range from 0.0001 to 0.0006, but sometimes as much as 0.0010 [20]. Empirical methods for the estimation of shrinkage strains have been proposed by ACI Committee [21].

When the total tensile stresses caused by shrinkage and external loads exceed the strength of the concrete, cracking will occur. These cracks, however, are undesired in high-strength concrete, especially for some important RC structures with high requirements on sealing performance or chemical corruption (*e.g.* water-retaining structures, nuclear reactor vessels and offshore oil platforms). In addition, volume contraction of RC members due to shrinkage will induce unpredictable forces to adjoining members in a large structural system. As a consequence, the control on shrinkage becomes essential in the analysis and design of concrete, and arouses extensive attention of engineers. It is recognized that the reduction of shrinkage can be achieved by the rational process of cement hydration, the ambient relative humidity and the correct positioning of reinforcements [22].

In contrast to the aforementioned publications, the goal of this study is to determine the effect of shrinkage in the optimal RC layout design with high control requirements on cracking and crushing. The shrinkage of concrete is numerically simulated by introducing an additional pressure force and the concrete strength yield surface is represented by two Drucker–Prager functions. Based on the solid isotropic material with penalization (SIMP) model, the topology optimization problem is defined as to minimize the costs of steel reinforcements under strength yield constraints on concrete elements and a shrinkage volume constraint. The enhanced aggregation strategy for an efficient solving of the large-scale strength constrained problem is used along with the adjoint-variable based sensitivity analysis. In the numerical examples, a comparison with the layout results without considering shrinkage is provided and the applicability of the proposed method is also demonstrated.

The article is organized as follows. An overview of the finite element analysis based on the SIMP formulation is given in Section 2, including the simulation of the concrete shrinkage. The Drucker–Prager criterion used for concrete failure surface is introduced in Section 3. Sections 4 and 5 present the emphasis of this article, where the optimization problem formulation, the solution strategy and the sensitivity analysis are presented. Several numerical examples are presented in Section 6, and conclusions are provided in Section 7.

2. Shrinkage of concrete and finite element analysis

Given a design domain Ω_{des} meshed by N finite elements, the SIMP scheme introduces a continuous variable $\rho_e \in [0,1]$ for each element. Considering RC structures built from two candidate materials (steel and concrete) without void, the relative densities for the concrete phase and the steel phase in the eth element are represented by $\rho_e^{\text{c}} = 1 - \rho_e$ and $\rho_e^{\text{s}} = \rho_e$, respectively. The elasticity tensor of each element is thus related to the variable ρ_e and defined by [23]

$$\mathbf{D}_e = (1-\rho_e^p)\mathbf{D}_0^{\text{c}} + \rho_e^p\mathbf{D}_0^{\text{s}} \quad (e = 1,2,\ldots,N) \tag{1}$$

where $\mathbf{D}_0^{\text{c}}$ and $\mathbf{D}_0^{\text{s}}$ are the elasticity tensor of pure concrete material and that of pure steel material, respectively. The power p must be larger than one to impose a penalization, but not be too large for satisfying the so-called Hashin–Shtrikman bounds [24]. Usually we choose $p = 3$.

Assume the used concrete has a free shrinkage strain ε_{sh}, the unrestrained strain state of elemental concrete phase is represented as

$$\boldsymbol{\varepsilon}_0 = \varepsilon_{\text{sh}}[1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]^{\text{T}} \tag{2}$$

The shrinkage of concrete can be numerically simulated by introducing an additional force on the RC structure, which is given by

$$\mathbf{F}^{\text{ad}} = \sum_{e=1}^{N}\mathbf{f}_e^{\text{ad}}, \quad \mathbf{f}_e^{\text{ad}} = -\int_{V_e}\mathbf{B}_e^{\text{T}}\mathbf{D}_e^{\text{c}}\boldsymbol{\varepsilon}_0\,dV = -(1-\rho_e^p)\int_{V_e}\mathbf{B}_e^{\text{T}}\mathbf{D}_0^{\text{c}}\boldsymbol{\varepsilon}_0\,dV \tag{3}$$

where $\mathbf{B}_e$ is the strain–displacement matrix of element e, $\mathbf{D}_e^{\text{c}} = (1-\rho_e^p)\mathbf{D}_0^{\text{c}}$ is the elastic tensor for concrete phase.

The force displacement relation corresponding to the shrinkage is then expressed as

$$\mathbf{K}\mathbf{u}^{\text{ad}} = \mathbf{F}^{\text{ad}} \tag{4}$$

where $\mathbf{K}$ is the global stiffness matrix and $\mathbf{u}^{\text{ad}}$ is the structural displacement due to the shrinkage.

Considering the shrinkage strains are small, the relative change in volume of the RC structure is given by

$$\begin{aligned} S &= \frac{\Delta V}{V_0} = \frac{1}{V_0}\sum_{e=1}^{N}V_e\theta_e = \frac{1}{V_0}\sum_{e=1}^{N}V_e(\varepsilon_{e,1}+\varepsilon_{e,2}+\varepsilon_{e,3}) \\ &= \frac{1}{V_0}\sum_{e=1}^{N}V_e\mathbf{w}\boldsymbol{\varepsilon}_e^{\text{ad}} = \frac{1}{V_0}\sum_{e=1}^{N}V_e\mathbf{w}\mathbf{B}_e\mathbf{u}_e^{\text{ad}} \end{aligned} \tag{5}$$

where $\mathbf{w} = [1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]$. The volumetric strain θ_e represents the relative change per unit volume [25]. $\varepsilon_{e,1}, \varepsilon_{e,2}$ and $\varepsilon_{e,3}$ are the principal strains of the eth element. $\boldsymbol{\varepsilon}_e^{\text{ad}}$ and $\mathbf{u}_e^{\text{ad}}$ are the elemental strain and the element displacement due to shrinkage, respectively. V_e is the element volume and V_0 is the total structural volume. The relative change in volume S is also called the "*volume shrinkage ratio*". Obviously, if the structure is unconstrained and consists of fully plain concrete, the volume shrinkage ratio S reaches its maximal value $S_{\max}$.

The stress level of the element due to shrinkage can be deduced by an initial strain analysis. Fig. 1 shows the combination of the shrunk concrete phase and the un-shrunk steel phase in an intermediate density element. Note that no stress exists in both material phases if they are separated. Strain changes and stress takes place after combined, accompanied by the mutual restraint between the steel phase and the concrete phase. The change in strain of the concrete phase after combined is

$$\Delta\boldsymbol{\varepsilon}_e^{\text{c}} = \boldsymbol{\varepsilon}_e - \boldsymbol{\varepsilon}_0 = \mathbf{B}_e\mathbf{u}_e^{\text{ad}} - \boldsymbol{\varepsilon}_0 \tag{6}$$