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Dynamic misspecification in nonparametric cointegrating regression

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ABSTRACT

Linear cointegration is known to have the important property of invariance under temporal translation. The same property is shown not to apply for nonlinear cointegration. The limit properties of the Nadaraya–Watson (NW) estimator for cointegrating regression under misspecified lag structure are derived, showing the NW estimator to be inconsistent, in general, with a "pseudo-true function" limit that is a local average of the true regression function. In this respect nonlinear cointegrating regression differs importantly from conventional linear cointegration which is invariant to time translation. When centred on the pseudo-true function and appropriately scaled, the NW estimator still has a mixed Gaussian limit distribution. The convergence rates are the same as those obtained under correct specification ($\sqrt{h\sqrt{n}}$, h is a bandwidth term) but the variance of the limit distribution is larger. The practical import of the results for index models, functional regression models, temporal aggregation and specification testing are discussed. Two nonparametric linearity tests are considered. The proposed tests are robust to dynamic misspecification. Under the null hypothesis (linearity), the first test has a χ^2 limit distribution while the second test has limit distribution determined by the maximum of independently distributed χ^2 variates. Under the alternative hypothesis, the test statistics attain a $h\sqrt{n}$ divergence rate.

1. Introduction

Arguably, all econometric models abstract from reality and are potentially misspecified in uncertain ways. Even if the form of an econometric model were to accurately characterise reality, there is still a myriad of ways in which the generating mechanism for the observed data can depart from the posited model. Therefore, it is important to know the limit properties of various estimators when the underlying model is misspecified. A series of papers in the econometric and statistics literature attempts to cast light on this problem. See for example Berk (1966, 1970), Domowitz and White (1982), Gourieroux et al. (1984), Huber (1967), White (1981, 1982) *inter alia*. Some of the questions raised by the aforementioned papers are summarised by White (1982):

"*If one does not assume that the probability model is correctly specified, it is natural to ask what happens to the properties of the* [*maximum likelihood*] *estimator. Does it still converge to some limit asymptotically, and does this limit have any meaning? If the estimator is somehow consistent, is it also asymptotically normal?*"

It is well known that, under certain conditions, parametric estimators of stationary misspecified models have a well defined limit referred to in the econometric literature as a *pseudo-true value*.[1] The asymptotic analysis of misspecified models is not only of theoretical interest. To obtain asymptotic power rates for various specification tests e.g. Bierens (1990), Ramsey (1969) (or tests without a specific alternative) knowledge about the asymptotic behaviour of the estimator under misspecification is necessary. Moreover, to determine the limit distribution of certain model selection statistics under the null hypothesis, e.g. Cox (1961, 1962), Davidson and McKinnon (1981) and Voung (1989) (tests

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[1] The pseudo-true value can be different than the parameter of interest and is determined by the value that optimises a certain limit criterion function (see for example Huber, 1967; Akaike, 1973; White, 1982; Bierens, 1984).

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with a specific alternative), the estimator's limit distribution about the pseudo-true value, is required.

The current paper takes the Wang and Phillips (2009a, hereafter WP)[2] framework and analyses the effects of misspecification relating to the lag structure of a cointegrating model. Further, two nonparametric linearity tests that are robust to dynamic misspecification are proposed. This kind of dynamic misspecification is potentially relevant in a variety of contexts and is especially germane in situations where temporal aggregation issues arise. We show that the consequences of dynamic misspecification in a nonstationary framework largely depend on the nature of the regression function and on the nature of the functions involved in the estimation procedure (see Theorem 1 and Example 1). The current work also relates to Phillips (2009) and Kasparis (2011). Phillips (2009) analyses spurious nonparametric regression, while Kasparis (2011) considers the effects of functional form misspecification in the presence of stochastic trends.

One of the main results of the present paper is to show that the Nadaraya–Watson (NW) kernel estimator under dynamic misspecification exhibits inconsistency in nonstationary regression due to the use of integrable functions in the construction of the kernel regression function. It will be shown that, under certain regularity conditions, the effect of the lag misspecification is to induce a shift in the limit, based on a local average of the function around each regression point (i.e. the NW estimator has a pseudo-true function limit). This kind of behaviour is similar to the limit in the case of misspecified dynamics in a stationary time series setting. In this respect, we find analogous results for dynamically misspecified nonparametric models between stationary and nonstationary cases. On the other hand, there is a big difference between nonlinear cointegration models where dynamic mistiming induces inconsistency, as shown here, and linear cointegration models where consistency continues to hold under dynamic mistiming.

The NW estimator, when centred on the pseudo-true function and appropriately scaled, has a mixed Gaussian limit distribution. The convergence rates are the same as those reported by WP. Nevertheless, the variance of the limit distribution is larger than that obtained under correct specification. We also consider the case of severe dynamic misspecification where the lag differential between the true and the fitted models is large. For badly misspecified models, the limit theory is substantially different. In this case, the NW estimator may be divergent, vanish or converge to a limit involving some stochastic integral.

This kind of dynamic-induced inconsistency arises in many other cases where the model and estimation procedure involves integrable functions and timing issues are relevant in specification. For example, the maximum likelihood estimator of discrete choice models involves integrable functions (see Park and Phillips, 2000) and will be similarly subject to the effects of dynamic specification error. Issues of timing in dynamic specification are likely to be particularly important in market intervention models of the type studied in Hu and Phillips (2004).

Moreover, two linearity tests that are robust to dynamic misspecification are proposed. The test statistics under consideration involve a comparison of the NW kernel estimator with parametric least squares. Asymptotic properties of the tests are derived. Under the null hypothesis of linearity, the first test has a χ^2 limit distribution while the second test has a limit distribution determined by the maximum of independently distributed χ^2 variates. The tests are consistent against integrable and locally integrable alternatives. The divergence rate is of order $h\sqrt{n}$.

[2] Wang and Phillips (2009a) provide a limit theory for nonparametric cointegrating regression. For related work see Guerre (2004), Karlsen et al. (2007), Schienle (2008) and Wang and Phillips (2009b, 2011).

The remainder of the paper is organised as follows. Section 2 provides limit theory for kernel regression under dynamic misspecification. Section 3 provides some applications in contexts of interest for applied work. Section 4 develops linearity tests that are robust to dynamic misspecification. The finite sample properties of the linearity tests are explored in a simulation experiment. Section 5 concludes. Technical results and proofs are given in the Appendices. Notation is fairly standard. For instance, we use $a \vee b$ ($a \wedge b$) to denote the maximum (minimum) of two real numbers a and b, and $=_d$ represents distributional equality. Throughout the paper summations such as $\sum_{t\geq 1}^{n}$ are interpreted as sums over $1 \leq r \vee s \vee l \leq n$ whenever there are integer parameters such as r, s, l governing the initialisation. Finally, I denotes the integrable family of functions, and LI denotes the locally integrable family of functions, that are not integrable.

2. Kernel regression under dynamic misspecification

This section develops a limit theory for the Nadaraya–Watson kernel regression estimator in the case of dynamic misspecification. It is well known (e.g. White, 1981, 1982; Domowitz and White, 1982) that, under certain regularity conditions, parametric estimators of misspecified models converge to some well defined pseudo-true value that is typically different than the parameter of interest. In the current paper it is demonstrated that, when the fitted model suffers from dynamic misspecification, and under certain regularity conditions, the NW estimator has a well defined limit. When the dynamic misspecification is mild — that is, the lag differential between the true models is finite — the NW has a *pseudo-true function* limit. The pseudo-true function corresponds to the true regression function as long as the latter is linear. In general the pseudo-true function differs from the true function and is determined by some local average of the true regression function. If dynamic misspecification is severe in the sense that the lag differential between the true and fitted models goes to infinity in large samples there is no pseudo-true function limit. In this case, the NW diverges, vanishes or converges to some random limit, depending on the properties of the true regression function.

Next, we specify the model under consideration. Throughout the paper, we assume that the time series $\{y_t\}_{t=1}^{n}$ is generated by the model:

$$y_t = f(x_{t-r}) + u_t, \quad \text{for some integer lag } r \geq 0, \tag{1}$$

where f is a locally integrable regression function. The variable x_t is a nonstationary process defined on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$. For example, in many applications it will be sufficient for $\{x_t\}_{t=1}^{n}$ to be generated as a unit root process or as a near integrated array of the commonly used form

$$x_t = \rho_n x_{t-1} + v_t, \quad x_0 = 0, \tag{2}$$

where v_t is some error term whose properties are specified later (Assumptions 2.2 and 2.3 below) and $\rho_n = 1 - \frac{c_o}{n}$ for some constant c_o. To avoid unnecessary triangular array complications in the development that follows we focus on the unit root generating model for x_t, although our main results continue to hold with minor changes under (2). The regression error u_t is a martingale difference sequence. Both x_t and u_t are defined on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$. The exact properties of f, x_t and u_t will be specified in detail later.

We concentrate on the case where a version of (1) is fitted by nonparametric kernel regression. However, the fitted model involves a lag misspecification resulting from incorrect timing, so that the fitted model has the (lag misspecified) form

$$y_t = \hat{f}(x_{t-s}) + \hat{u}_t, \quad \text{for some fixed integer lag } s \geq 0,\ r \neq s, \tag{3}$$