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Inverse finite element modeling of shells using the degenerate solid approach

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ABSTRACT

The Inverse Finite Element Method (IFEM) for degenerate solid shells is introduced. IFEM allows determining the undeformed shape of a body (in this case, a shell-like body) such that it attains a desired shape after large elastic deformations. The model is based on the degenerate solid approach, which enables the use of the standard constitutive laws of Solid Mechanics. First, IFEM is applied to three popular benchmarks for validation purposes. Then, the capabilities of IFEM for inverse design are demonstrated by means of its application to the design of a microvalve.

1. Introduction

The Inverse Finite Element Method (IFEM) is the Finite Element Method (FEM) applied to the problem of determining the undeformed configuration of a body when the deformed configuration as well as the actuating loads are known. This kind of problem – also known as the Inverse Design problem – often arises in the design of compliant structures or mechanisms suffering large elastic displacements and/or rotations, for instance: a gasket that deforms to the desired shape under given loads [1]; a rubber seal that closes a given channel under a given pressure [2]; a turbine blade that attains an optimal shape at a certain angular speed [3]; an S-clutch whose shoes exactly engage the friction surface of a given drum at a given angular speed [4,5]; a device that folds an intraocular lens in such a way that facilitates its implantation into the eye [6], among other interesting applications developed in the papers mentioned.

Outside the field of inverse design, Lu and Zhou [7,8] proposed an application of IFEM to the prevention of aneurysms, taking the *in vivo* image of an aneurysm as the known deformed configuration under a known pressure.

All these inverse problems could be solved using systematized "trial-and-error" methods from Optimization Theory, considering any measure of the closeness to the desired deformed configuration as the cost function to be minimized. At each iteration of the optimization problem, a nonlinear (direct) equilibrium equation has to be solved to determine the cost function. In contrast, IFEM solves only one nonlinear equilibrium equation to determine the desired deformed configuration, which is approximately as computationally expensive as only one iteration of an optimization problem. This was illustrated by Albanesi et al. [4,5], who used IFEM to design a compliant gripper, which had been originally designed by Lan and Cheng [9] by solving an optimization problem.

In our previous papers, IFEM was introduced for 3D solids [3] and 3D beams [4,5]. The current paper is a step towards the completion of our IFEM library by introducing shell elements.

Zhou and Lu [8] introduced IFEM for shells using the stress-resultant approach proposed by Simo et al. [10]. Models based in this approach need specialized constitutive equations for the accross-the-thickness membrane and shear stress resultants and stress couple, as described in the pioneering work of Simo and Fox [11].

In the present paper, the degenerate solid approach to shells, originally proposed by Ahmad et al. [12] and extended to nonlinear geometrical analysis by Ramm [13], is preferred. This approach is characterized by defining the stress itself (rather than the stress resultants) using the same constitutive equations as those of Solid Mechanics. This attribute of the degenerate solid shells was the reason for our choice. Then, as an original contribution, we introduce IFEM to the context of degenerate solid shells.

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The low-order displacement-based shell finite elements predict spurious shear stresses and, as result, exhibit artificially high stiffness. This is the well-known "shear locking" defect [14], which can be circumvented by using the appropriate mixed finite elements. In the present paper, recourse is made to the formulation known as Mixed Interpolation of Tensorial Components (MITC), originally proposed by Dvorkin and Bathe [15] for bi-linear 4-node quadrangles and extended by Bucalem and Bathe [16] to bi-quadratic 9-node and bi-cubic 16-node quadrangles. Using MITC, the components of the strain tensor are interpolated independently of the displacements, in order to preclude shear locking.

First, we solve three popular problems for linear-elastic shells with large deflections and rotations [17], using these benchmarks for the purpose of validating the presented IFEM. Finally, the capability of IFEM for inverse design is shown by the design of a compliant microvalve to close a given channel when the pressure drop attains a prescribed value, giving a more efficient alternative to that originally proposed by Seidemann et al. [18].

2. Formulation of the degenerate solid shell finite element

The aim of this section is to give a brief summary of the formulation of FEM for degenerate solid shells, which is already classical in the "direct" FEM. Specifically, we describe the so-called "basic shell" model [19,20], which is based on the Mindlin–Reissner kinematic hypothesis: those straight fibers that are normal to the midsurface of the shell when it is undeformed remain straight and unstretched during deformation. The "basic shell" model is well-suited for thin to moderately thick shells, offering the best compromise between simplicity and applicability in FEM for shells.

As a corollary, we arrive at a system of discrete nonlinear equations governing the equilibrium of geometrically nonlinear degenerate solid shells in "direct" FEM, to be taken as the starting point for the development of IFEM for degerate solid shells in the next section.

2.1. Kinematic hypotheses for shells

Let $\mathcal{B}^0$ represent the solid shell body shown in Fig. 1. The geometry of the shell is defined by its midsurface $\mathcal{S}^0$ and the thickness of the shell at each point of the midsurface. Let $\{\xi_1, \xi_2, \xi_3\}$ be a system of natural coordinates, such that ξ_1 and ξ_2 vary through the midsurface $\mathcal{S}^0$ and ξ_3 varies across the thickness of the shell, with $-1 \leqslant \xi_i \leqslant 1$ and $\xi_3 = 0$ at the midsurface. Then, the position of any point $\boldsymbol{X} \in \mathcal{B}^0$ can be expressed as a function of the natural coordinates as follows:

$$\boldsymbol{X}(\xi_1, \xi_2, \xi_3) = \overline{\boldsymbol{X}}(\xi_1, \xi_2) + \xi_3 \frac{H}{2} \boldsymbol{T}(\xi_1, \xi_2), \quad (1)$$

where $\overline{\boldsymbol{X}} \in \mathcal{S}^0, \boldsymbol{T}$ is the unit vector known as the material director, and $H = H(\xi_1, \xi_2)$ is the thickness of the undeformed shell.

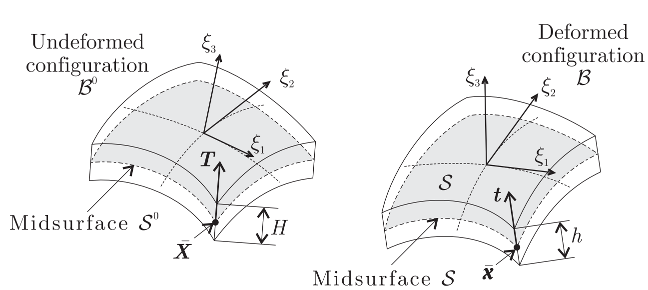

Fig. 1. Geometric representation of the undeformed and deformed configurations of a shell.

Let $\mathcal{B}$ be the deformed configuration of the shell, with midsurface $\mathcal{S}$. After deformation, the point $\boldsymbol{X} \in \mathcal{B}^0$ occupies the position $\boldsymbol{x} \in \mathcal{B}$:

$$\boldsymbol{x}(\xi_1, \xi_2, \xi_3) = \bar{\boldsymbol{x}}(\xi_1, \xi_2) + \xi_3 \frac{h}{2} \boldsymbol{t}(\xi_1, \xi_2), \quad (2)$$

where $\bar{\boldsymbol{x}} \in \mathcal{S}, \boldsymbol{t}$ is the unit vector known as the spatial director, and $h = h(\xi_1, \xi_2)$ is the thickness of the undeformed shell.

In the present paper, we adopt the "basic shell" model [19,20], based on the Mindlin–Reissner plate theory, according to which $\boldsymbol{t}$ is not necessarily normal to $\mathcal{S}$ if $\boldsymbol{T}$ is normal to $\mathcal{S}^0$ (and viceversa), as an effect of the shear deformation. Further, as a consequence of the Mindlin–Reissner assumption, the strain normal to the midsurface is null [20], so that the thickness of the shell remains constant during deformation, i.e., $h = H$.

Inside a generic finite element with nodes $i = 1, 2, \ldots, N$, the positions $\boldsymbol{x} \in \mathcal{B}$ and $\boldsymbol{X} \in \mathcal{B}^0$ are isoparametrically interpolated from their respective nodal values, as follows:

$$\boldsymbol{X}(\xi_1, \xi_2, \xi_3) = \varphi_i(\xi_1, \xi_2)\left[\overline{\boldsymbol{X}}_i + \frac{\xi_3}{2} h(\xi_1, \xi_2) \boldsymbol{T}_i\right] = \boldsymbol{\Phi}(\xi_1, \xi_2, \xi_3)\boldsymbol{Q}, \quad (3)$$

$$\boldsymbol{x}(\xi_1, \xi_2, \xi_3) = \varphi_i(\xi_1, \xi_2)\left[\bar{\boldsymbol{x}}_i + \frac{\xi_3}{2} h(\xi_1, \xi_2) \boldsymbol{t}_i\right] = \boldsymbol{\Phi}(\xi_1, \xi_2, \xi_3)\boldsymbol{q}, \quad (4)$$

with

$$\boldsymbol{\Phi} = \left[\, \varphi_1 \boldsymbol{I}_{3\times3} \quad \tfrac{\xi_3}{2} h \varphi_1 \boldsymbol{I}_{3\times3} \quad \cdots \quad \varphi_N \boldsymbol{I}_{3\times3} \quad \tfrac{\xi_3}{2} h \varphi_N \boldsymbol{I}_{3\times3} \,\right], \quad (5)$$

$$\boldsymbol{Q} = \begin{bmatrix} \overline{\boldsymbol{X}}_1 \\ \boldsymbol{T}_1 \\ \vdots \\ \overline{\boldsymbol{X}}_N \\ \boldsymbol{T}_N \end{bmatrix}, \qquad \boldsymbol{q} = \begin{bmatrix} \bar{\boldsymbol{x}}_1 \\ \boldsymbol{t}_1 \\ \vdots \\ \bar{\boldsymbol{x}}_N \\ \boldsymbol{t}_N \end{bmatrix}, \quad (6)$$

where $(\overline{\boldsymbol{X}}_i, \boldsymbol{T}_i)$ defines the position of node i in the finite element mesh representing $\mathcal{B}^0$ (known for FEM, unknown for IFEM), $(\bar{\boldsymbol{x}}_i, \boldsymbol{t}_i)$ defines the position of node i in the mesh representing $\mathcal{B}$ (unknown for FEM, known for IFEM), and $\varphi_i = \varphi_i(\xi_1, \xi_2)$ is the 2-D shape function associated to node i; $\boldsymbol{I}_{3\times3}$ is the 3×3 identity matrix.

The deformation of the shell can be measured using the Green–Lagrange strain tensor, which can be expressed as

$$\boldsymbol{E} = \underbrace{\frac{1}{2}(\boldsymbol{g}_\alpha \cdot \boldsymbol{g}_\beta - \boldsymbol{G}_\alpha \cdot \boldsymbol{G}_\beta)}_{E^{\mathrm{cov}}_{\alpha\beta}} \boldsymbol{G}^\alpha \otimes \boldsymbol{G}^\beta, \quad (7)$$

where $E^{\mathrm{cov}}_{\alpha\beta}$ are the so-called covariant components of $\boldsymbol{E}$, $\boldsymbol{g}_\alpha = \partial\boldsymbol{x}/\partial\xi_\alpha$ and $\boldsymbol{G}_\alpha = \partial\boldsymbol{X}/\partial\xi_\alpha$ are the spatial and convective basis vectors, respectively, and $\boldsymbol{G}^\alpha$ is a vector of the base reciprocal to $\{\boldsymbol{G}_\alpha\}$, so that $\boldsymbol{G}^\alpha \cdot \boldsymbol{G}_\beta = \delta^\alpha_\beta$.

Using FEM, the covariant strain components $E^{\mathrm{cov}}_{\alpha\beta}$ take the form

$$E^{\mathrm{cov}}_{\alpha\beta} = \frac{1}{2}\left(\boldsymbol{q}^T \boldsymbol{A}_{\alpha\beta} \boldsymbol{q} - \boldsymbol{Q}^T \boldsymbol{A}_{\alpha\beta} \boldsymbol{Q}\right), \quad (8)$$

where $\boldsymbol{A}_{\alpha\beta}$ is the $6N \times 6N$-symmetric matrix defined by

$$\boldsymbol{A}_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial\boldsymbol{\Phi}^T}{\partial\xi_\alpha}\frac{\partial\boldsymbol{\Phi}}{\partial\xi_\beta} + \frac{\partial\boldsymbol{\Phi}^T}{\partial\xi_\beta}\frac{\partial\boldsymbol{\Phi}}{\partial\xi_\alpha}\right). \quad (9)$$

2.2. The cure for shear locking

The stiffness of low-order finite elements increases spuriously as the thickness/in-plane dimension of the element decreases. This is the well-known "shear locking" problem, which affects even cubic order elements.