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A novel state space method for force identification based on the Galerkin weak formulation

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ABSTRACT

A novel force identification method in state space based on the Galerkin weak formulation is proposed. Using the time function interpolations of both displacement and force, the equation of motion of structural system in state space based on the principle of weighted momentum is formulated. The method is more suitable for the cases of large time step and discontinuous loading compared with the conventional state space method and the explicit Newmark method. Additionally, the proposed method is extended to a refined version for the case of high noise level by dividing the time step into several smaller time substeps.

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1. Introduction

Accurate knowledge of the dynamic force on a structural system during operation is vital in many structural dynamic problems, such as structural dynamic design, response reconstruction, condition assessment and health monitoring. Unfortunately, in many practical applications, a direct measurement of the input forces is very difficult, especially the interaction forces between different substructures of a large coupled structural system. In some situations, if force gauges are inserted into force transfer path to measure the excitations, it is difficult to obtain the accurate forces since the existence of the force gauges may alter the system properties. And also, an impact force with large magnitude for a short time period is difficult to be measured. Instead, vibration responses can be conveniently measured. The indirect method of force identification, which is to perform an inverse identification process with the measured structural responses and the known structural system, is often used in practice. The force identification is an important topic in structural dynamics. Existing force identification methods can be mainly classified into two categories, the frequency domain [1–6] and time domain methods [7–14]. The frequency domain methods of estimating forces were originally developed in the 1980s using the frequency response functions. The forces are identified in frequency domain. To obtain the forces in time domain, inverse Fourier transform is required. The inverse transform makes those methods suitable for stable and stationary random forces, but difficult to identify impact transient forces. The time domain methods identify the forces directly in time domain, which have no transform errors and can be suitable for any types of forces.

Most existing time domain force identification methods are based on the state space method, which relates the structural responses, the system parameters and the input forces with the Markov parameters. Law and Fang [15] presented a state space method for moving force identification. Kammer [16] and Law et al. [17] adopted the zeroth order system Markov parameters to identify the external forces. Nordberg and Gustafsson [18] proposed an explicit block inversion algorithm to invert the associated upper block triangular Toeplitz matrix for force identification using the state space method. Mao et al. [19] established a precise force identification model based on the precise time-step integration and Tikhonov regularization technique. Law and Yong [20] used the state space method to identify the interface forces and the external forces for structural condition assessment. Recently, a novel approach of force identification based on average acceleration discrete algorithm was presented in the state space [21]. Although the state space method is not sensitive to the initial values and have no cumulative errors produced by the previous time step compared with many other time domain force identification method, it still has an obvious disadvantage. The method is explicit and assumed that the force is a constant value in each discrete time step. Therefore, the method is accurate only when the time step is small enough.

To overcome this disadvantage, the conventional implicit Newmark method for the forward dynamic analysis was transformed into an equivalent explicit form for force identification [22]. The explicit Newmark algorithm is more accurate compared

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with the conventional state space method, especially when the sampling frequency is low. However, most of the existing direct integration techniques [23,24] including the Newmark method require very small time steps to be used in dealing with the time history response of a structure subjected to a discontinuous loading [25–29], such as a rectangular impulse or sequential triangular impulses. This is because the dynamic excitation in the time domain shows discontinuities or rapid changes at the end of the impulse-type force. Chang [25] pointed out that using the principle of momentum to transform a second order equation of motion into a first order momentum equation of motion is beneficial to smooth out the rapid changes in dynamic loading.

In this paper, a novel force identification method in state space is proposed based on the Galerkin weak (GW) formulation which was originally presented to compute the forward problem of structural dynamics [30]. The conventional second order equation of motion of structural system is transformed into a set of first order equations of motion in state space based on the principle of momentum. A recurrence matrix equation in terms of the state vector of displacement and velocity over the time step is formulated. The force identification equation is built with the recurrence matrix and measurement equation as the same as the conventional state space method for force identification. The proposed method is very suitable for the cases of large time step and discontinuous loading.

In addition, a large noise can lead to producing a serious error of the identified force. The identification method which can deal with the circumstance of large noise is valuable in practice. However, very little literature on the aspects of large noise is found. In order to improve the accuracy of force identification in the case of large noise, the proposed GW method is refined based on the discretization idea of the Finite Element Method (FEM). The time step of measured responses is divided into several smaller time substeps to form an overdetermined identification equation. Therefore, more response information in the refined GW method is utilized to identify the unknown forces compared with that in the GW method, which is beneficial to improve the identification accuracy.

This paper proceeds as follows. Section 2 presents the GW method for force identification and the refined version for the case of high noise level. In Section 3, numerical studies are conducted to demonstrate the effectiveness of the proposed method in comparison to the conventional state space method and the Newmark method. Effects of measurement noise, multiple forces and discontinuity force reconstruction are also considered in detail. Finally, several conclusions are drawn based on the current study. In Appendix A, the stability and accuracy of the refined GW method is illustrated. Appendix B presents the detailed matrices of the two cases of the refined GW method, including the cases of the two time subintervals and four time subintervals.

2. The proposed method

The general equation of motion of a damped structure with n DOFs can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the structure, respectively; $\ddot{\mathbf{x}}(t), \dot{\mathbf{x}}(t), \mathbf{x}(t)$ are, respectively, the nodal acceleration, velocity and displacement vectors of the structure; $\mathbf{f}(t)$ is the force vector. Rayleigh damping $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ is assumed in this paper, where α and β are the Rayleigh damping coefficients. However, there is no limitation on the type of damping model adopted in the proposed method.

2.1. The GW method for force identification

The weighted residual approach was adopted to obtain the generalized Newmark method with the time FE techniques presented by Zienkiewiez [31,32]. Selecting different weighted function can obtain different time integration formulation. Xing et al. [30] proposed a time integration method based on the weak form Galerkin method. In this paper, the method is transformed to an explicit form for force identification.

Referring to the discretization of space FE in structural dynamics, the time domain is discretized into N time elements. The time interval is denoted as $t_n \leqslant t \leqslant t_{n+1}$, and the time step is Δt. The displacement $\mathbf{x}(t)$ over the time step Δt can be approximated as

$$\mathbf{x}(t) = \mathbf{N}^T\mathbf{d} \tag{2}$$

where $\mathbf{N}$ is the interpolation function, $\mathbf{d}$ is the vector of nodal displacements of the time element, and the superscript T represents the transpose operation. The weighted residual method is employed to convert the original Eq. (1) into an integral form with the time interval of $t_n \leqslant t \leqslant t_{n+1}$. By multiplying Eq. (1) by a weight function $w_i(t)$, and integrating the equation over the time step from t_n to t_{n+1}, one can obtain the following integral equation as

$$\int_{t_n}^{t_{n+1}} w_i(\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) - \mathbf{f}(t))dt = 0, \quad i = 1, 2, \ldots \tag{3}$$

Here, the weight functions are valued as the same as the interpolation functions, which is called the Galerkin method. Eq. (3) can be rewritten as

$$\int_{t_n}^{t_{n+1}} \mathbf{N}\mathbf{M}\ddot{\mathbf{x}}(t)dt + \int_{t_n}^{t_{n+1}} \mathbf{N}\mathbf{C}\dot{\mathbf{x}}(t)dt + \int_{t_n}^{t_{n+1}} \mathbf{N}\mathbf{K}\mathbf{x}(t)dt - \int_{t_n}^{t_{n+1}} \mathbf{N}\mathbf{f}(t)dt = 0 \tag{4}$$

The first term of Eq. (4) can be computed by the subsection integration as follows

$$\begin{aligned}\int_{t_n}^{t_{n+1}} \mathbf{N}\mathbf{M}\ddot{\mathbf{x}}(t)dt &= [\mathbf{N}\mathbf{M}\dot{\mathbf{x}}(t)]_{t_n}^{t_{n+1}} - \int_{t_n}^{t_{n+1}} \dot{\mathbf{N}}\mathbf{M}\dot{\mathbf{x}}(t)dt \\ &= [\mathbf{N}\mathbf{M}\mathbf{v}(t)]_{t_n}^{t_{n+1}} - \int_{t_n}^{t_{n+1}} \dot{\mathbf{N}}\mathbf{M}\dot{\mathbf{x}}(t)dt\end{aligned} \tag{5}$$

where $\mathbf{v}(t)$ is the vector of velocity. Substituting Eqs. (2) and (5) into Eq. (4), and transforming integration interval from $[t_n, t_{n+1}]$ into $[-1, 1]$, one can obtain as

$$\mathbf{K}^e\mathbf{d} + \frac{2}{\Delta t}[\mathbf{N}(1) \otimes \mathbf{M}\mathbf{v}_{n+1} - \mathbf{N}(-1) \otimes \mathbf{M}\mathbf{v}_n] = \mathbf{f}^e \tag{6}$$

where $\mathbf{K}^e = \mathbf{L}^k \otimes \mathbf{K} + \frac{2}{\Delta t}\mathbf{L}^c \otimes \mathbf{C} - \frac{4}{\Delta t^2}\mathbf{L}^m \otimes \mathbf{M}, \otimes$ represents the Kronecker product; and $\mathbf{L}^k = \int_{-1}^{1} \mathbf{N}\mathbf{N}^T d\xi$; $\mathbf{L}^c = \int_{-1}^{1} \mathbf{N}\dot{\mathbf{N}}^T d\xi$; $\mathbf{L}^m = \int_{-1}^{1} \dot{\mathbf{N}}\dot{\mathbf{N}}^T d\xi$; $\mathbf{f}^e = \int_{-1}^{1} \mathbf{N} \otimes \mathbf{f}(\xi)d\xi$; and $\xi = \frac{2t - t_n - t_{n+1}}{\Delta t}$, $-1 \leqslant \xi \leqslant 1$, where ξ represents the natural coordinate, which is a nondimensional time parameter.

The external force function can be also approximated by the interpolation function, then $\mathbf{f}^e = (\mathbf{L}^k \otimes \mathbf{I})\mathbf{p}$, where $\mathbf{p}$ is the vector of nodal force of the time element. Two time nodes are assumed in each time element and linear interpolation is employed. The linear interpolation functions are denoted as

$$N_1(\xi) = \frac{1}{2}(1 - \xi) \quad N_2(\xi) = \frac{1}{2}(1 + \xi) \tag{7}$$

Substituting Eq. (7) into Eq. (6), one can obtain

$$\mathbf{K}^e \begin{Bmatrix} \mathbf{d}_n \\ \mathbf{d}_{n+1} \end{Bmatrix} + \frac{2}{\Delta t} \begin{Bmatrix} 0 \\ \mathbf{M}\mathbf{v}_{n+1} \end{Bmatrix} - \frac{2}{\Delta t} \begin{Bmatrix} \mathbf{M}\mathbf{v}_n \\ 0 \end{Bmatrix} = \mathbf{f}^e \tag{8}$$

Expanding Eq. (8), the two following equations can be obtained as

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