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A micromechanical-statistical model based on hypersingular boundary integral equations for analyzing a pair of parallel interfaces weakened by antiplane micro-cracks

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ABSTRACT

The estimation of the effective stiffness coefficients of a pair of microscopically damaged interfaces in a trimaterial under antiplane deformations is considered here. The trimaterial is made of a thin elastic layer sandwiched between two elastic half-spaces. The parallel planar interfaces are modeled as containing periodic arrays of randomly generated micro-cracks. The micromechanical-statistical model of the interfaces is formulated and numerically solved in terms of hypersingular boundary integral equations in which the displacement jumps across the micro-cracks are unknown functions. The numerical results obtained from the model demonstrate that the effective stiffness coefficients are influenced by the elastic moduli of the trimaterial, the thickness of the elastic layer and the densities of the micro-cracks.

1. Introduction

Multilayered structures play an increasingly important role in many engineering applications. Bonded layers of materials may be created by chemical or physical deposition processes [13]. During such processes, residual stresses may be induced by mismatches in the elastic or thermal properties of the different materials, giving rise to the formation of micro-cracks on the interface between two dissimilar materials [18]. Microscopic gaps may also exist on the interface because of the micro-roughness of surfaces. Thus, in general, the different layers in a multilayered structure are imperfectly bonded.

At the macro-level, an interface weakened by micro-defects may be modeled as a spring-like imperfect interface characterized by a stiffness tensor. In the macroscopic model, the displacement field is discontinuous across the spring-like interface and the tractions on the interface are linearly related to the displacement jumps over the interface [4,11].

Many researchers have studied boundary value problems involving the macro spring-like interface (see, for example, [2,9,10]). There are, however, relatively few studies on the micromechanical estimation of the effective stiffness of the imperfect interface. Fan and Sze [8] presented a finite element based three-phase model for estimating the electric conduction coefficient of a micro-cracked interface between two dielectric half-spaces. The three-phase model takes into consideration only the density of the interfacial micro-cracks. To model the interface more realistically, Wang et al. [16] proposed a micromechanical-statistical approach in which the sizes and positions of the micro-cracks are randomly generated.

In [16], a selected number of micro-cracks of varying sizes are randomly generated and positioned to form a finite interval of the microscopically damaged interface between two elastic half-spaces. The interval containing the micro-cracks is periodically reproduced on the remaining parts of the interface. The micromechanical-statistical model in [16] is formulated and solved in terms of hypersingular integral equations. In such a hypersingular integral formulation, the unknown functions are the displacement jumps over micro-cracks on the interface [3]. Thus, no post processing is needed to compute the interface displacement jumps which are required in estimating the effective stiffness of the interface. In [16], the micro-crack length is assumed to follow a normal distribution. For a more realistic distribution of the micro-crack sizes, Wang et al. [17] employed a chi-square (χ^2) distribution of a low degree of freedom to generate the micro-crack length.

The estimation of the effective stiffness coefficients of a pair of microscopically damaged interfaces in a trimaterial under antiplane deformations is considered in the current paper. The trimaterial is made of a thin elastic layer sandwiched between two

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elastic half-spaces. The micromechanical-statistical approach in [16,17] is used to model the two parallel planar interfaces. The resulting boundary value problem is formulated in terms of hypersingular boundary integral equations. Once the hypersingular boundary equations are solved, the effective stiffness coefficients of the interfaces may be readily computed. The effects of the elastic moduli of the trimaterial, the thickness of the elastic layer and the densities of the micro-cracks on the effective stiffness coefficients of the two interfaces are investigated.

2. A micromechanical problem of a pair of micro-cracked interfaces

With reference to a Cartesian coordinate system $Ox_1x_2x_3$, a thin elastic layer occupies the region $0 < x_2 < h$ and is sandwiched between two elastic half-spaces in the regions $x_2 < 0$ and $x_2 > h$. The interfaces $x_2 = 0$ and $x_2 = h$ between the layer and the half-spaces are microscopically damaged, containing interfacial micro-cracks. The micro-cracks have geometries independent of the x_3 coordinate. For convenience, the micro-cracked interfaces $x_2 = 0$ and $x_2 = h$ are denoted by I and II respectively.

The materials in the layer and half-spaces are anisotropic, having possibly dissimilar elastic properties. The trimaterial is assumed to undergo an antiplane elastostatic deformation along the x_3 direction. The x_1 and x_2 components of the elastic displacement are zero and the x_3 component, denoted by u_3, is a function of x_1 and x_2 only. According to Hooke's Law, the antiplane stresses σ_{3j} $(j = 1, 2)$ are related to the spatial derivatives of u_3 by

$$\sigma_{3j} = \lambda_{ij}(x_2)\frac{\partial u_3}{\partial x_i}, \tag{1}$$

where $\lambda_{ij}(x_2)$ are elastic moduli of the anisotropic materials given by

$$\lambda_{ij}(x_2) = \begin{cases} \lambda_{ij}^{(1)} & \text{for } x_2 > h, \\ \lambda_{ij}^{(2)} & \text{for } 0 < x_2 < h, \\ \lambda_{ij}^{(3)} & \text{for } x_2 < 0, \end{cases} \tag{2}$$

with $\lambda_{ij}^{(p)}$ being constants such that $\lambda_{ij}^{(p)} = \lambda_{ji}^{(p)}$ and $\lambda_{11}^{(p)}\lambda_{22}^{(p)} - (\lambda_{12}^{(p)})^2 > 0$. The usual Einsteinian convention of summing over a repeated index is assumed here for only Latin subscripts from 1 to 2.

From (1) and the equilibrium equations of elastostatics, the antiplane displacement u_3 satisfies the elliptic partial differential equation

$$\frac{\partial}{\partial x_j}(\lambda_{ij}(x_2)\frac{\partial u_3}{\partial x_i}) = 0. \tag{3}$$

If the displacement u_3 and stress σ_{3j} along a macroscopic portion of the microscopically damaged interface I or interface II is homogenized using

$$\begin{aligned} \widehat{u}_3(\widehat{x}_1, 0^{\pm}) &= \frac{1}{2\ell}\int_{\widehat{x}_1-\ell}^{\widehat{x}_1+\ell} u_3(x_1, 0^{\pm})dx_1, \\ \widehat{u}_3(\widehat{x}_1, h^{\pm}) &= \frac{1}{2\ell}\int_{\widehat{x}_1-\ell}^{\widehat{x}_1+\ell} u_3(x_1, h^{\pm})dx_1, \\ \widehat{\sigma}_{3j}(\widehat{x}_1, 0^{\pm}) &= \frac{1}{2\ell}\int_{\widehat{x}_1-\ell}^{\widehat{x}_1+\ell} \sigma_{3j}(x_1, 0^{\pm})dx_1, \\ \widehat{\sigma}_{3j}(\widehat{x}_1, h^{\pm}) &= \frac{1}{2\ell}\int_{\widehat{x}_1-\ell}^{\widehat{x}_1+\ell} \sigma_{3j}(x_1, h^{\pm})dx_1, \end{aligned} \tag{4}$$

where $\widehat{x}_1$ and ℓ denote the midpoint and length of the macroscopic portion respectively, then the boundary conditions for the macro-level spring-like model for the interfaces are given by (see [16,10])

$$\begin{aligned} \widehat{k}_{\mathrm{I}}\Delta\widehat{u}_{3\mathrm{I}}(\widehat{x}_1) &= \widehat{\sigma}_{32}(\widehat{x}_1, 0^+) = \widehat{\sigma}_{32}(\widehat{x}_1, 0^-), \\ \widehat{k}_{\mathrm{II}}\Delta\widehat{u}_{3\mathrm{II}}(\widehat{x}_1) &= \widehat{\sigma}_{32}(\widehat{x}_1, h^+) = \widehat{\sigma}_{32}(\widehat{x}_1, h^-), \end{aligned} \tag{5}$$

where $\Delta\widehat{u}_{3\mathrm{I}}(\widehat{x}_1) = \widehat{u}_3(x_1, 0^+) - \widehat{u}_3(x_1, 0^-)$ and $\Delta\widehat{u}_{3\mathrm{II}}(\widehat{x}_1) = \widehat{u}_3(x_1, h^+) - \widehat{u}_3(x_1, h^-)$ are the displacement jumps across interfaces I and II respectively and $\widehat{k}_{\mathrm{I}}$ and $\widehat{k}_{\mathrm{II}}$ denote the effective stiffness of interfaces I and II respectively. The two interfaces are assumed to be homogeneous at the macro level, hence the effective stiffness coefficients $\widehat{k}_{\mathrm{I}}$ and $\widehat{k}_{\mathrm{II}}$ are constant.

Note that the antiplane stress $\widehat{\sigma}_{32}$ in (5) is the antiplane traction on the micro-cracks. This is because the micro-cracks lie on the horizontal planes $x_2 = 0$ and $x_2 = h$.

At the microscopic level, the two interfaces are modeled as containing periodical arrays of interfacial micro-cracks. For a simplified model, each of the interfaces contains M arbitrarily positioned micro-cracks of possibly different lengths lying on a period interval of the interface of length L. On the part of interface I where $0 < x_1 < L$, the tips of a typical m-th micro-crack are given by $(a_{\mathrm{I}}^{(m)}, 0)$ and $(b_{\mathrm{I}}^{(m)}, 0)$, where $a_{\mathrm{I}}^{(m)}$ and $b_{\mathrm{I}}^{(m)}$ are constants such that $0 < a_{\mathrm{I}}^{(1)} < b_{\mathrm{I}}^{(1)} < a_{\mathrm{I}}^{(2)} < b_{\mathrm{I}}^{(2)} < \cdots < a_{\mathrm{I}}^{(M)} < b_{\mathrm{I}}^{(M)} < L$. On the remaining parts of interface I, the interfacial micro-cracks lie at where $a_{\mathrm{I}}^{(m)} + nL < x_1 < b_{\mathrm{I}}^{(m)} + nL$, for $m = 1, 2, \ldots, M$ and $n = \pm 1, \pm 2, \ldots$. The part of interface II where $L_0 < x_1 < L_0 + L$ contains M micro-cracks with tips $(a_{\mathrm{II}}^{(m)}, 0)$ and $(b_{\mathrm{II}}^{(m)}, 0)$ (for $m = 1, 2, \ldots, M$), where L_0 is a given positive number such that $0 \leqslant L_0 < L$ and $a_{\mathrm{II}}^{(m)}$ and $b_{\mathrm{II}}^{(m)}$ are such that $L_0 < a_{\mathrm{II}}^{(1)} < b_{\mathrm{II}}^{(1)} < a_{\mathrm{II}}^{(2)} < b_{\mathrm{II}}^{(2)} < \cdots < a_{\mathrm{II}}^{(M)} < b_{\mathrm{II}}^{(M)} < L_0 + L$. The micro-cracks on interface II outside where $L_0 < x_1 < L_0 + L$ lie in the regions $a_{\mathrm{II}}^{(m)} + nL < x_1 < b_{\mathrm{II}}^{(m)} + nL, x_2 = 0$, for $m = 1, 2, \ldots, M$ and $n = \pm 1, \pm 2, \ldots$. Fig. 1 gives a sketch of the geometry of the problem for $M = 3$. The uncracked parts of the interfaces are perfectly bonded.

The damage ratios of interfaces I and II are respectively defined by

$$\rho_{\mathrm{I}} = \frac{1}{L}\sum_{m=1}^{M}(b_{\mathrm{I}}^{(m)} - a_{\mathrm{I}}^{(m)}) \text{ and } \rho_{\mathrm{II}} = \frac{1}{L}\sum_{m=1}^{M}(b_{\mathrm{II}}^{(m)} - a_{\mathrm{II}}^{(m)}). \tag{6}$$

From (6), because of the assumption that the two interfaces have the same number of micro-cracks, the damage ratios can be shown to be related to the average half lengths of the micro-cracks on the

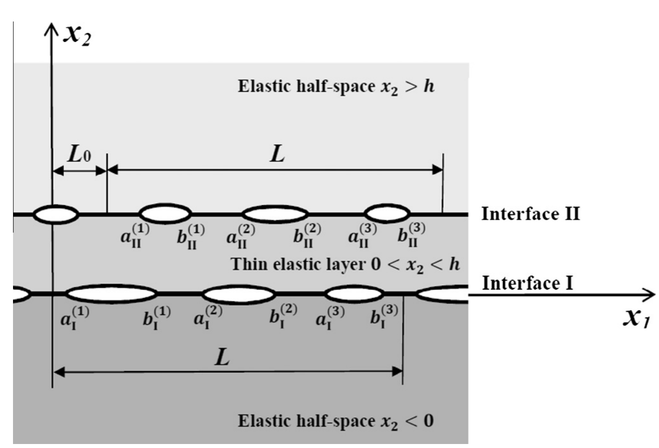

Fig. 1. A sketch of the geometry of the problem for $M = 3$.