Computers and Structures 157 (2015) 189–200

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

The local response in structures using the Embedded Unit Cell Approach

Marina Grigorovitch, Erez Gal *

Department of Structural Eng., Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

ARTICLE INFO

Article history:
Received 2 April 2015
Accepted 5 May 2015
Available online 10 June 2015

Keywords:
Embedded Unit Cell (EUC)
Multiscale formulation
Non-periodic unit cell
Local–global analysis
Stress concentration
Finite element method

ABSTRACT

This paper presents the development of a new concept, the Embedded Unit Cell (EUC) approach, to calculate local responses in elastic media. The EUC approach is based on a multi-scale formulation of non-periodic domains to evaluate the local/micro response where stress concentrations are expected. The formulation is based on alternative boundary conditions which is not restricted to periodic assumptions of the unit cell response that is required in the classical theory. This approach provides a reduced computational cost model of the macroscopic/global problem while preserving the accuracy at the micro-scale problem. We conclude with a numerical verification study.

1. Introduction

Multiscale modeling is one of the most powerful engineering tools to evaluate the mechanical response of composite-material structures [1–13]. In this type of analysis the micro-scale properties of the composite materials are homogenized and micro-scale post processing is required to obtain the micro-scale response, e.g., see [14–24]. The generalized theory of homogenization [25–30], which provides the mathematical formulation of multi scale analysis [24,31–38], is based on the assumptions of a periodic unit cell (microscopic) response [39–43]. In this paper, we suggest the embedded unit cell (EUC) approach which circumvents the restrictions imposed by the period response assumption. This enables the use of multi-scale technics to obtain the local response in stress concentration engineering problems also refered as local global problems.

Numerous theories have been developed to predict the behavior of composite materials with various effective properties methods e.g. see [44–46]. Most of these analytical models can only give estimates or boundaries for the macroscopic properties for complex microstructures. In addition, computational procedures for implementing homogenization are active areas of research e.g. see [30,47]. These developments have established the Finite Element Method (FEM) [47–55] as one of the most efficient numerical methods, whereby the macroscopic responses can be obtained by volumetrically averaging numerical solutions of unit cells e.g. see [56]. All these contributions assume periodic response of the micro-scale (unit cell) and therefore cannot be utilized for local global problems where periodic do not exist.

Multi scale formulation which circumvents the periodic assumption has been suggested by Fish and Fan [57]. Fish and Fan [57] suggested a formulation based on an asymptotic expansion of the displacement field around an arbitrary reference point. They used this formulation to obtain the microscopic large displacement response and also discuss its ability to predict the response in local–global engineering problems having non-periodic response. Other investigation of the non-periodic problems have been discussed by Allaire [34] and Cherdantsev [39], which showed that two-scale finite element multi-scale analysis convergence can be achieved without periodicity. The proposed research intends to establish multi-scale technics based on the theory of homogenization for stress concentration engineering problems.

In this research we are suggesting the EUC multi-scale approach in order to obtain the local response in stress concentration engineering problems (local global problems). In the suggested formulation we are adjusting the theory of homogenization to be used in a domain decomposing scheme. By that we are dividing the stress concentration problem into two independent models which are coupled using a unit cell having alternative boundary conditions by mean of an interface zone (surrounding domain) between the local (microscopic) and the global (macroscopic) models. By that instead of solving one demanding problem we are solving two much less demanding problems while preserving the accuracy. The formulation is based on the asymptotic homogenization theory without periodic assumption by suggesting alternative boundary conditions. Finally, we conclude with a numerical implementation and verification study.

* Corresponding author.
E-mail address: erezgal@bgu.ac.il (E. Gal).

http://dx.doi.org/10.1016/j.compstruc.2015.05.006
0045-7949/

2. Embedded Unit Cell formulation

To obtain the local response in various engineering problems, domain decomposition techniques are usually required e.g. see [58–60]. These approaches refine the mesh in the vicinity of the local domains and thus result in a model that includes a large number of degrees of freedoms [61]. In contrast, the suggested EUC formulation uses several sequential models, i.e., one for the "macroscopic" problem which does not include the refinement in the vicinity of the stress concentration and "microscopic" problems which describe only the domain at the vicinity of the stress concentration.

The EUC approach is based on a non-periodic multiscale formulation with alternative boundary conditions. We are suggesting achieving that by surrounding the stress concentration domain (unit cell) with an appropriate continuum domain. This surrounding domain appropriately accounts for the boundary conditions, required to represent the response at the vicinity of the stress concentration domain using a unit cell model (micro-scale model). In the suggested EUC approach, we are using this unit cell models in a multi-scale scheme to homogenize the stress concentration domain (unit cell models) and up-scale its properties to the macro-scale (structural) model. The following sections describe the mathematical formulation of the suggested EUC approach.

2.1. The theory of homogenization

Let assume, that macroscopic body Ω^ζ is formed by heterogeneous microscopic structure with local periodicity set by unit cell [62–66] see Fig. 1.

We assume that the problem is defined by two scales, global (coarse) scale, D, order of the macroscopic body Ω^ζ and local (fine) scale, d, order of the microscopic unit cell.

The relation between the global coordinate system x_i to the local coordinate system y_i is defined by:

$$y_i = x_i/\zeta, \quad i = 1,2,3\ldots \tag{1}$$

where ζ is ratio between the scales such that or as:

$$\frac{\partial x_i}{\partial y_i} = \zeta\delta_{ij} \tag{2}$$

where δ_{ij} is the Kronecker Delta.

Response of heterogeneous body subjected to external loading, is described by physical measures, such as displacement u_i, strain ε_{ij} and stress σ_{ij}, that change slowly at the global coordinates and change rapidly at microscopic coordinate y at the vicinity ζ of a given point x.

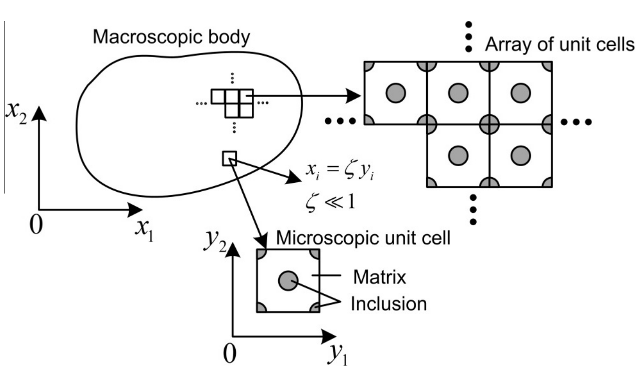

Fig. 1. Heterogeneous microscopic structure with local periodicity unit cells.

So $u_i, \varepsilon_{ij}, \sigma_{ij}$ depend on both coarse and fine scales, as:

$$\begin{aligned} u_i^\zeta &= u_i^\zeta(x,y) \\ \varepsilon_{ij}^\zeta &= \varepsilon_{ij}^\zeta(x,y) \\ \sigma_{ij}^\zeta &= \sigma_{ij}^\zeta(x,y) \end{aligned} \tag{3}$$

where $(i,j) = 1,2$ for two-dimensional problem, and $(i,j) = 1,2,3$ for three-dimensional problem. The superscript ζ denotes periodicity of the physical measures, such that:

$$\begin{aligned} u_i^\zeta(x,y) &= u_i^\zeta(x,y+kY) \\ \varepsilon_{ij}^\zeta(x,y) &= \varepsilon_{ij}^\zeta(x,y+kY) \\ \sigma_{ij}^\zeta(x,y) &= \sigma_{ij}^\zeta(x,y+kY) \end{aligned} \tag{4}$$

where k denotes specific unit cell index and $Y(y_k)$ denote the unit cell size.

Solutions of the unknown parameters such as displacement u_i, the mechanic strain ε_{ij} and stress σ_{ij} are achieved by using the following set of Eqs. (5)–(9) and boundary conditions for displacements and tractions (see also [60]).

$$\frac{\partial \sigma_{ij}^\zeta}{\partial x_j} + f_i = 0 \quad in\ \Omega^\zeta \tag{5}$$

Kinematical:

$$\varepsilon_{ij}^\zeta = \frac{1}{2}\left(\frac{\partial u_i^\zeta}{\partial x_j} + \frac{\partial u_j^\zeta}{\partial x_i}\right) \quad in\ \Omega^\zeta \tag{6}$$

Constitutive equation:

$$\sigma_{ij}^\zeta = C_{ijkl}^\zeta(\varepsilon_{kl}^\zeta - \mu_{kl}) + \lambda_{ij} \quad in\ \Omega^\zeta \tag{7}$$

Boundary conditions – essential:

$$u_i^\zeta = \bar{u}_i \quad on\ \Gamma_u \tag{8}$$

Boundary conditions – natural:

$$\sigma_{ij}^\zeta n_j = \bar{t}_i \quad on\ \Gamma_\sigma \tag{9}$$

The material properties tensor C_{ijkl}^ζ is symmetric with respect to indices $(i,j,k,l), f_i$ represents body forces, μ_{kl}, λ_{ij} are initial stress and strain respectively. n_j are the outward normal unit vectors to $\partial\Omega$, the boundary of the domain Ω. $\partial\Omega$ is composed of the segments Γ_u and Γ_σ, on which the displacements and tractions boundary conditions are defined.

The displacement field $u_i^\zeta(x,y)$ can be described by the following asymptotic expansion:

$$u_i^\zeta(x,y) = u_i^{(0)}(x,y) + \zeta u_i^{(1)}(x,y) + \zeta^2 u_i^{(2)}(x,y) + \cdots \tag{10}$$

For any function $f(x,y)$, the derivative can be defined by the chain rule as:

$$\frac{d}{dx_j} = \frac{\partial}{\partial x_j} + \frac{\partial}{\partial y_k}\frac{\partial y_k}{\partial x_j} \tag{11}$$

So that derivatives of the displacement field can be described as follows: $\frac{du_i^\zeta(x,y)}{dx_j} = \frac{\partial u_i^\zeta(x,y)}{\partial x_j} + \frac{\partial u_i^\zeta(x,y)}{\partial y_k}\frac{\partial y_k}{\partial x_j}$.

Substituting Eq. (2) into Eq. (11), we obtain that:

$$\frac{du_i^\zeta(x,y)}{dx_j} = \frac{\partial u_i^\zeta(x,y)}{\partial x_j} + \frac{1}{\zeta}\frac{\partial u_i^\zeta(x,y)}{\partial y_j} = \frac{\partial u_i^\zeta(x,y)}{\partial x_j} + \zeta^{-1}\frac{\partial u_i^\zeta(x,y)}{\partial y_j} \tag{12}$$