Journal of Econometrics 158 (2010) 285–305

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Applications of subsampling, hybrid, and size-correction methods

Donald W.K. Andrews [a,*], Patrik Guggenberger [b]

[a] Cowles Foundation for Research in Economics, Yale University, United States

[b] Department of Economics, UCSD, United States

ARTICLE INFO

Article history:
Available online 18 January 2010

JEL classification:
C12
C15

Keywords:
Asymptotic size
Finite-sample size
Hybrid test
Instrumental variable
Over-rejection
Parameter near boundary
Size correction
Subsampling confidence interval
Subsampling test
Weak instrument

ABSTRACT

This paper analyzes the properties of subsampling, hybrid subsampling, and size-correction methods in two non-regular models. The latter two procedures are introduced in Andrews and Guggenberger (2009a). The models are non-regular in the sense that the test statistics of interest exhibit a discontinuity in their limit distribution as a function of a parameter in the model. The first model is a linear instrumental variables (IV) model with possibly weak IVs estimated using two-stage least squares (2SLS). In this case, the discontinuity occurs when the concentration parameter is zero. The second model is a linear regression model in which the parameter of interest may be near a boundary. In this case, the discontinuity occurs when the parameter is on the boundary.

The paper shows that in the IV model one-sided and equal-tailed two-sided subsampling tests and confidence intervals (CIs) based on the 2SLS t statistic do not have correct asymptotic size. This holds for both fully- and partially-studentized t statistics. But, subsampling procedures based on the partially-studentized t statistic can be size-corrected. On the other hand, symmetric two-sided subsampling tests and CIs are shown to have (essentially) correct asymptotic size when based on a partially-studentized t statistic. Furthermore, all types of hybrid subsampling tests and CIs are shown to have correct asymptotic size in this model. The above results are consistent with "impossibility" results of Dufour (1997) because subsampling and hybrid subsampling CIs are shown to have infinite length with positive probability.

Subsampling CIs for a parameter that may be near a lower boundary are shown to have incorrect asymptotic size for upper one-sided and equal-tailed and symmetric two-sided CIs. Again, size-correction is possible. In this model as well, all types of hybrid subsampling CIs are found to have correct asymptotic size.

1. Introduction

This paper continues the investigation initiated in Andrews and Guggenberger (2009a; 2009b; forthcoming) (hereafter denoted AG2, AG3, and AG1) of the properties of subsampling and subsampling-based procedures in non-regular models. We apply the results of AG1–AG3 to two models. The first model is an instrumental variables (IVs) regression model with possibly weak IVs. This is a leading example of a broad class of models in which lack of identification occurs at some point(s) in the parameter space. It is a model that has been studied extensively in the recent econometrics literature. For this reason, it is a natural model to use to assess the behavior of subsampling methods. The second example that we consider in this paper concerns a CI when the parameter of interest may be near a boundary. This example is a generalization of the example used in the introduction of AG1 to illustrate heuristically a problem with subsampling. Here we treat the example rigorously.

In the first example, for comparability to the literature, we focus on a model with a single right-hand-side (rhs) endogenous variable and consider inference concerning the parameter on this variable. It is well-known that standard two-stage least squares (2SLS) based t tests and CIs have poor size properties in this case, e.g., see Dufour (1997), Staiger and Stock (1997), and references cited therein. In particular, one-sided, symmetric two-sided, and equal-tailed two-sided fixed critical value (FCV) tests have finite-sample size of 1.0. Furthermore, these tests cannot be size-corrected by increasing the FCV.[1]

* Corresponding address: Cowles Foundation for Research in Economics, Yale University, P.O. Box 208281, Yale Station, New Haven, CT 06520-8281, United States. Tel.: +1 203 432 3698; fax: +1 203 432 6167.

E-mail address: donald.andrews@yale.edu (D.W.K. Andrews).

[1] The finite-sample (or exact) *size* of a test is defined to be the maximum rejection probability of the test under distributions in the null hypothesis. A test is said to have *level* α if its finite-sample size is α or less. The *asymptotic size* of a test is defined to be the limit superior of the finite-sample size of the test. The finite-sample (or exact) *size* of a confidence interval (or confidence set) is defined to be the minimum coverage probability of the confidence interval under distributions in the model.

0304-4076/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.jeconom.2010.01.002

We are interested in the properties of subsampling methods in this model. We are also interested in whether the hybrid and size-correction (SC) methods introduced in AG2 can be used to obtain valid inference in this well-known non-regular model. Hence, we investigate the size properties of subsampling and hybrid tests based on the 2SLS estimator. The test results given here apply without change to CIs (because of location invariance). We also consider size-corrected versions of these methods. Alternatives in the literature to the size-corrected methods include the conditional likelihood ratio (CLR) test of Moreira (2003), the rank CLR test of Andrews and Soares (2007), and the adaptive CLR test of Cattaneo et al. (2007). These tests are asymptotically similar, and hence, have good size properties. Also, their power properties have been shown to be quite good in Andrews et al. (2006, 2007, 2008) and the other references above. Other tests in the literature that are robust to weak IVs include those given in Kleibergen (2002, 2005), Guggenberger and Smith (2005, 2008), and Otsu (2006). Although we have not investigated the power properties of the hybrid and SC subsampling tests considered here, we expect that they are inferior to those of the CLR, rank CLR, and adaptive CLR tests. Hence, we do not advocate the use of subsampling methods in the weak IV model for inference on the parameter of a single rhs endogenous variable.

However, the CLR-based tests and the other tests mentioned above do not apply to inference concerning the parameter on one endogenous variable when multiple rhs endogenous variables are present that may be weakly identified. This is a testing problem for which no asymptotically similar test is presently available. The methods analyzed in this paper are potentially useful for such inference problems. We leave this to future research.

We now summarize the results for the IV example. We show that subsampling tests and CIs do not have correct size asymptotically, but can be size-corrected. The asymptotic rejection probabilities of the subsampling tests are found to provide poor approximations to the finite-sample rejection probabilities in many cases. But, the finite-sample adjusted asymptotic rejection probabilities introduced in AG2 perform very well across all scenarios. In consequence, the adjusted size-corrected subsampling (ASC-Sub) tests perform well. For example, the nominal 5% ASC-Sub tests based on partially-studentized t statistics have finite-sample sizes of 4.4%, 5.3%, and 4.4% for upper one-sided, symmetric two-sided, and equal-tailed two-sided tests in a model with $n = 120$, $b = 12$, 5 IVs, and normal errors.

The hybrid test is found to have correct size asymptotically and very good size in finite samples for upper one-sided and symmetric two-sided tests—4.8% and 4.7%, respectively. For equal-tailed two-sided tests, the hybrid test has correct size asymptotically, but is conservative in finite samples. For the same parameter values as above, the nominal 5% hybrid test has finite-sample size of 2.8%.

We show that nominal $1 - \alpha$ subsampling CIs have infinite length with probability $1 - \alpha$ asymptotically when the model is completely unidentified and the correlation between the structural and reduced-form errors is ± 1. This holds for both fully- and partially-studentized t statistics. This result is of particular interest given Dufour's (1997) result that the 2SLS CI based on a fixed critical value, and any CI that has finite length with probability one, have a finite-sample size of zero for all sample sizes. The results given in this paper are consistent with those of Dufour (1997) and explain why size-correction of subsampling procedures is possible even in the presence of lack of identification at some parameter values.

Analogously, a confidence interval is said to have *level* $1 - \alpha$ if its finite-sample size is $1 - \alpha$ or greater. The *asymptotic size* of a confidence interval is defined to be the limit inferior of the finite-sample size of the confidence interval. A test is called *asymptotically similar* if the limit of the null rejection probability of the test is the same under any sequence of nuisance parameters.

In the second example we consider a multiple linear regression model where the regression parameter of interest θ ($\in R$) is restricted to be non-negative. We consider a studentized t statistic based on the least squares estimator of θ that is censored to be non-negative.

The results for this example are summarized as follows. Lower one-sided, symmetric two-sided, and equal-tailed two-sided subsampling CIs for θ based on the studentized t statistic do not have correct asymptotic coverage probability. In particular, these three nominal $1 - \alpha$ CIs have asymptotic confidence levels of $1/2$, $1 - 2\alpha$, and $(1 - \alpha)/2$, respectively. Hence, the lower and equal-tailed subsampling CIs perform very poorly in terms of asymptotic size. The finite-sample sizes of these tests are found to be close to their asymptotic sizes in models with ($n = 120$, $b = 12$) and ($n = 240$, $b = 24$) and normal errors and regressors. Size-correction is possible for all three types of subsampling CIs. The SC subsampling CIs are found to have good size in finite samples, but display a relatively high degree of non-similarity. The upper one-sided subsampling CI has correct asymptotic size $1 - \alpha$.

We show that all types of FCV and hybrid CIs have correct asymptotic size—no size correction is necessary. These CIs are found to have finite-sample sizes that are fairly close to their nominal sizes. The FCV CIs exhibit the smallest degree of finite-sample non-similarity, which has CI length advantages. Hence, somewhat ironically, the best CIs in this example are FCV CIs that ignore the presence of a boundary. We caution, however, that the scope of this result is limited to CIs when a scalar parameter of interest may be near a boundary and no other parameters are.

Using results in the literature, such as Andrews (1999, 2001), the asymptotic results given here for subsampling, FCV, and hybrid CIs should generalize to a wide variety of models other than regression models in which one or more parameters may be near a boundary.

The Appendix of the paper provides necessary and sufficient conditions for size-correction (of the type considered in AG2) to be possible in the general set-up considered in AG1 and AG2.

Literature that is related to this paper include AG1 and AG2, as well as Politis and Romano (1994) and Politis et al. (1999). Andrews and Guggenberger (2009c) discusses an additional example regarding the performance of subsampling methods. Somewhat related is the paper by Moreira et al. (2009) on bootstrapping the CLR test in an IV regression model with possibly weak IVs.

The remainder of this paper is organized as follows. Section 2 summarizes the most relevant results in AG1 and AG2 to make the paper more self-contained. Section 3 discusses the IV regression example. Section 4 discusses the regression example in which the parameter of interest may be near a boundary. An Appendix contains the verifications of assumptions in AG1 and AG2, including proofs of the asymptotic distributions of t statistics in these examples. The Appendix also provides the necessary and sufficient conditions for size-correction to be possible.

2. Summary of AG1 and AG2

The treatment of the two examples considered in Sections 3 and 4 relies heavily on the theoretical results on the "asymptotic size" of a test given in AG1 and AG2. To make the paper more self-contained and easier to read, we summarize in this section some of the most relevant results of AG1 and AG2. We illustrate and motivate the assumptions and theoretical results in AG1 and AG2 through a simplified version of the weak IV example of Section 3. We also provide a brief discussion of the relevance of the two examples considered in Sections 3 and 4.