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Robust topology optimization of truss structures with random loading and material properties: A multiobjective perspective

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ABSTRACT

In this paper an approach to robust topology optimization for truss structures with material and loading uncertainties, and discrete design variables, is investigated. Uncertainties on the loading, and spatially correlated material stiffness, are included in the problem formulation, taking truss element length into account. A more realistic random field representation of the material uncertainties is achieved, compared to classical scalar random variable approaches. A multiobjective approach is used to generate Pareto optimal solutions showing how the mean and standard deviation of the compliance can be considered as separate objectives, avoiding the need for an arbitrary combination factor.

1. Introduction

Truss structures are exceedingly common in engineering applications, including bridges, towers, in buildings, and numerous mechanical applications. Design optimization of these structures can lead to significant improvement of structural performance and material savings. In real-world truss structures uncertainty exists on material properties, geometry (such as manufacturing tolerances), and loading of structures and these can have significant impact on the performance of the structures designed using an optimization procedure. Structural optimization taking uncertainties into account is of importance to designers, since real-world structures require both efficient use of material and accurate modelling of material properties, manufacturing tolerances and loading of structures. When considering candidate topological designs, engineers are concerned with the sensitivity of the designs to small variations which can be quantified as uncertainties. In order to address this the concept of robust topology optimization (RTO) has become increasingly important in recent years, incorporating the variability of candidate solutions when considering the efficiency of that solution for dealing with a specific structural problem. In the last decade several important papers focussing on RTO have appeared. Kogiso et al. [1] used a sensitivity-based RTO for compliant mechanisms, with random variation on the loading direction. de Gournay et al. [2] investigated shape and topology optimization for minimal compliance, minimizing the 'worst case' compliance under perturbation of the loading. Guest and Igusa [3] used a mean compliance formulation under uncertainties on the nodal locations, while Lógó et al. [4] developed a new loading criterion for compliance minimization for probabilistic loading, and extended this to uncertainties on the loading location [5]. Chen et al. [6] proposed a robust shape and topology optimization (RSTO) method, taking material uncertainties into account. Tootkaboni et al. [7] developed a robust formulation for mass minimization with uncertainties on the material properties, using a polynomial chaos approach. Wang et al. [8] demonstrated a method for robust topology optimization applied to photonic waveguides, with manufacturing uncertainties. However, in spite of this recent research interest, several issues have been neglected:

1. The representation of random uncertainties in the literature is not always accurate, leading to incorrect quantification of the robustness of solutions.
2. Generally more than one type of uncertainty needs to be considered: material properties, loading, nodal positions, etc. The derivatives required for gradient-based methods become difficult or extremely complicated to define analytically in these circumstances.
3. Robust formulations of the topology optimization problem require a combination of two distinct quantities: mean and standard deviation. Often this is done through a virtually arbitrary linear combination of these response quantities.

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4. Truss structures contain bar elements in which the material properties vary along the length of a single element, and therefore cannot be dealt with in the same way as continuum structures.
5. Truss topology optimization problems often require discrete variable formulation [9].
Gradient-based algorithms are often poorly suited for addressing this class of problems.
6. Increasingly designers wish to take more than one objective into account in the optimization of real-world structures.
7. Designers are often concerned with objective functions which are broader than the classical compliance or mass functions. Such objectives generally do not lend themselves to classical gradient-based solutions and multiobjective methods of this kind have not been properly developed.

Starting from these considerations the paper is arranged as follows: the approach to the uncertainty modelling and the optimization approach is given in Section 2, followed by an explanation of the multiobjective approach in Section 3. Finally, several examples are given in Section 4 and conclusions discussed in Section 5.

2. Uncertainty quantification and optimization approach

The representation of uncertainties in robust optimization of truss structures has been relatively neglected and investigations thus far have failed to take some key features of trusses, such as element length, into account. Asadpoure et al. [10] developed a method for RTO of truss structures taking material stiffness uncertainties into account, assigning scalar uncertainties to each cross-section. Yonekura and Kanno [11] used semidefinite programming to solve truss robust topology problems with uncertainties on loading, while Kang and Bai [12] recently considered bounded uncertainties in truss structures.

On the other hand random fields allow for the expression of spatially varying material properties to be taken into consideration, and the stochastic finite element framework allows for integration of these fields into the structural analysis. The mean and standard deviations of the resulting structural responses can be extracted from this analysis and used in the definition of the objective function(s).

Within the robust formulation, it is required to take both first order (mean) and second (variance or standard deviation) order statistical moments of the structural response h into account. Generally a single-objective approach is adopted, considering the weighted sum of these two quantities:

$$\min_{\mathbf{x}} f(\mathbf{x}) = E[h(\mathbf{x})] + \alpha \cdot Std[h(\mathbf{x})] \tag{1}$$

However, particularly in the case of discrete variable problems, the choice of α may not be evident. For this purpose, it is of interest to consider the statistical moments of the response as separate objectives within a multiobjective framework, as done by Padovan et al. [13] for example.

In this investigation the uncertainties on the Young's modulus are expressed in terms of a spatially varying random field, which is discretized using a Karhunen–Loève (KL) expansion. Random fields allow for expression of spatially correlated random quantities, while being general enough to model uncorrelated quantities too. A novel application of Spectral Stochastic Finite Element Method (SSFEM) [14] is used for truss structures to derive the statistical measures of the response, allowing for a quantification of the terms of the objective functions and constraints. This method was developed by Richardson et al. [15], in which it was discussed how the framework could be extended to allow for introduction of loading uncertainties.

2.1. Material uncertainties

Material uncertainties are quantified in terms of probability distributions on values such as the Young's modulus. Material models are generally expressed in terms of Gaussian or lognormal probability distributions, both of which can be taken into account within the SSFEM framework. SSFEM discretization generally consists of series expansion methods, expanding any realization of the original random field $H(\mathbf{x}, \theta)$ over a complete set of deterministic functions [14], where θ is a vector of random variables. The obtained series are then truncated after a finite number of terms. Various discretization methods are available of which the KL expansion is the most efficient in terms of the number of random variables required for a given accuracy [14]. A Gaussian random field $H(\mathbf{x}, \theta)$ can be expanded as follows:

$$H(\mathbf{x}, \theta) = \mu(\mathbf{x}) + \sum_{i=1}^{\infty} \sqrt{\lambda_i}\xi_i(\theta)\varphi_i(\mathbf{x}) \tag{2}$$

where $\mu(\mathbf{x})$ is the mean value of the random phenomenon, λ_i's and φ_i's respectively the eigenvalues and eigenfunctions of the covariance kernel, and ξ_i's the random variables. The approximated field $\widehat{H}$ can be found by truncating terms above some value M:

$$\widehat{H} = \mu(\mathbf{x}) + \sum_{i=1}^{M} \sqrt{\lambda_i}\xi_i(\theta)\varphi_i(\mathbf{x}) \tag{3}$$

In continuum structures the random field may be correlated over the entire domain, however in truss structures this is not the case. Truss analysis accounting for material property uncertainty is often achieved by associating a random variable with the cross section area of each bar element. This is done analogously to the continuum topology optimization approaches, in which the domain is discretized, using a mesh of elements of the same size (as a rule). However, truss elements are generally not all the same length, and is depend on the mesh of the ground structure. This approach has two fundamental shortcomings:

1. The approach presupposes a small scale for the problem, while trusses and individual truss elements are typically large in scale, and
2. the relative lengths of the elements are neglected in the probabilistic model.

In the deterministic case, where uncertainties are not considered, this is of little importance, however, as will be seen, when uncertainties are taken into account, this can impact the robustness of the solution. At the scale of truss elements, often several meters in length, the variability of material properties along the length of the element can be very significant, spatially correlated quantities. Global 2D and 3D correlated random fields are not appropriate for modelling this variability, since no correlation exists between the material properties of separate elements. The proposed approach constructs individual 1D random fields across the individual truss elements, discretizing elements into sub-divisions. The analysis and topology variables apply to the truss scale elements and nodes (Fig. 1(a)). If each element is subdivided as shown in Fig. 1(a), a simple expression can be found to approximate the relative stiffness of the element as a whole, based on sampling the element-level field:

$$\widehat{H}_e = \frac{1}{\sum_{j=1}^{N_{se}} \frac{1}{\left(\mu_j(\mathbf{x}_j) + \sum_{i=1}^{M} \sqrt{\lambda_i}\xi_i(\theta)\varphi_i(\mathbf{x}_j)\right)}} \tag{4}$$

where $\widehat{H}_e$ is the element-level random field, μ_j is the mean value of the random field for sub-division j, and N_{se} is the number of