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# Modeling of fiber orientation in viscous fluid flow with application to self-compacting concrete

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ABSTRACT

In recent years, the application of Self Compacting Concrete (SCC) in industry is rapidly growing, which makes modeling of SCC casting an important part of the design process. The SCC is often combined with fiber reinforcement to deliver a high performance material, the properties of which depend on fiber orientation. Fiber reinforced SCC plays a fundamental role in reducing shrinkage effects in structures. The aim of this paper is to present a numerical model for the prediction of the fiber orientation state, determined on the basis of the knowledge of the velocity field. Statistical approach is employed, where orientation of a fiber is described by a probability distribution of the fiber angle, which evolves with the flow. Two ways how to solve the evolution of the probability distribution are presented. In the first way, evolution equation for the orientation distribution function (ODF) is derived and solved by FEM. In the second way, evolution equation for second-order orientation tensor, which represents the second statistical moment of ODF, is solved instead. The model is validated against experimental results.



## 1. Introduction

The modeling of fresh concrete flow is an interesting problem from both theoretical and practical points of view. Knowledge of fresh concrete behavior, flow abilities and casting capability have a significant importance, especially in connection with high-performance concrete (HPC) or self-compacting concrete (SCC). The SCC application is essential in highly reinforced structures where it is very hard to fill in all the voids as vibrating is not possible since there is limited space between the steel bars [1]. The design of fiber reinforced concrete assumes more or less uniform distribution of fibers, or at least fiber orientation in the direction of the principal stresses. When a material containing fibers is casted, its flow pattern changes the orientation of the fibers. It is obvious that the fiber orientation in the specimen is the key feature of its mechanical behavior. Fiber reinforced material is stronger and stiffer in the direction of the prevailing orientation and weaker and more compliant in the direction of the minor orientation. As there are methods to predict mechanical properties of the reinforced material once the orientation state is known (see for example [2]), the prediction of flow-induced fiber orientation remains a challenging task.

This paper presents a probabilistic based approach for predicting fiber orientation induced by fluid (SCC) flow, based on the work proposed in [3]. The model of fresh concrete flow is based on homogeneous approach, where the concrete suspension is considered as a single homogeneous medium and its motion in the Eulerian framework is described by the Navier–Stokes equations. The casting simulation in the context of the Eulerian formulation is typically modeled as a two-phase flow problem [4], where concrete and air are represented as two immiscible fluids separated by an interface, which has to be updated with the flow. The model has been described in [5].

The probabilistic approach is based on the assumption that the orientation state of the fiber can be completely described by the orientation probability distribution function. The evolution of probability distribution can be described by Fokker–Planck type of equation. The Fokker–Planck equation can be solved directly, but this is manageable only in two dimensional problems and for simple geometries, with a small amount of fibers. As an alternative, it is possible to expand distribution function into a series of its statistical moments and solve evolution equations for these moments, so called orientation tensors. The distribution function can be then reconstructed from these tensors, see [3,6,7]. Taking into account the statistical nature of the solved problem, it would be enough to know the direction of prevailing fiber orientation. As it will be shown in later sections, this information is provided by the

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statistical moment of second order. Instead of solving evolution equation for the complete probability distribution, it is enough to solve evolution equation only for the second order orientation tensor. Then, the eigenvectors of this orientation tensor represents the directions of prevailing and minor fiber orientations, while the eigenvalues represents the volume fraction of the fibers oriented in the direction of the corresponding eigenvector.

The paper is organized as follows. In Section 2 the governing equations describing the fluid flow are described. Strong and weak formulation of the problem are presented and the discretization using Finite Element Method is outlined. In Section 3, a probabilistic approach to fiber orientation tracking in fluid flow is presented. Two different approaches to solve the problem are described and discussed. Finally, in Section 4, the application of the developed model is presented on several examples. The model has been validated on the real experiment, comparing the numerical results with experimental fiber orientation form CT-scans.

## 2. Description of the flow

In this section, the governing Navier–Stokes equations of the flow in 2D will be presented. To start, consider a two-dimensional computational domain $\Omega \subset \mathbb{R}^2$ fully filled with two immiscible fluids, which occupy the portions $\Omega_1(t)$ and $\Omega_2(t)$. The boundary of the domain is denoted as $\partial\Omega$ and it can be decomposed into four mutually disjoint parts $\Gamma_D$, $\Gamma_N$, $\Gamma_{SWF}$ and $\Gamma_{OUT}$, forming complementary subsets of the boundary $\partial\Omega$, on which the Dirichlet, Neumann, so called "slip with friction" and "do nothing" boundary conditions are prescribed, respectively. The normal and tangent vectors on $\partial\Omega$ are denoted as $\boldsymbol{n}$ and $\boldsymbol{t}$, respectively. The interface between the two subsets filled with fluids, $\Omega_1(t)$ and $\Omega_2(t)$, is denoted as $\Sigma(t)$, and $\tilde{\boldsymbol{n}}$ is the normal vector on $\Sigma(t)$. Then, for each phase $j = 1,\ 2$ in the domain $\Omega$ and on its boundary $\partial\Omega$, the problem can be formulated as follows, see [8]

$$
\begin{aligned}
\rho_j\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \boldsymbol{b}\right) - \nabla\cdot\boldsymbol{\sigma} &= \boldsymbol{0} \text{ in } \Omega_j \times (0,\mathrm{T})\\
\nabla\cdot\boldsymbol{v} &= 0 \text{ in } \Omega_j \times (0,\mathrm{T})\\
\boldsymbol{v} &= \boldsymbol{g} \text{ on } \Gamma_D \times (0,\mathrm{T})\\
\boldsymbol{\sigma}\cdot\boldsymbol{n} &= \boldsymbol{h} \text{ on } \Gamma_N \times (0,\mathrm{T})\\
\boldsymbol{v}\cdot\boldsymbol{t} + \beta^{-1}\ \boldsymbol{n}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{t} &= 0 \text{ on } \Gamma_{SWF} \times (0,\mathrm{T})\\
\boldsymbol{n}\cdot\boldsymbol{\sigma} &= 0 \text{ on } \Gamma_{OUT} \times (0,\mathrm{T})\\
[\boldsymbol{v}]_{\Sigma(t)} &= 0 \text{ on } \Sigma \times (0,\mathrm{T})\\
[\boldsymbol{n}\cdot\boldsymbol{\sigma}]_{\Sigma(t)} &= 0 \text{ on } \Sigma \times (0,\mathrm{T})\\
\boldsymbol{v} &= \boldsymbol{v_0} \text{ in } \Omega_j, \quad t = 0.
\end{aligned}
\tag{1}
$$

Unknown fields are velocity $\boldsymbol{v}$ and pressure $p$. Density $\rho$, body forces $\boldsymbol{b}$ and functions $\boldsymbol{g}, \boldsymbol{h}$ and $\boldsymbol{v_0}$ are known. On the interface $\Sigma(t)$, conditions on jumps, denoted using square brackets $[\cdot]_{\Sigma(t)}$, of velocity and normal stress components are set. Note that in the case of fluid with surface tension, jump in normal stress would be proportional to the curvature of the interface $\Sigma(t)$.

Standard decomposition of stress tensor $\boldsymbol{\sigma}$ into deviatoric stress $\boldsymbol{\tau}$ and hydrostatic pressure $p$ is used

$$\boldsymbol{\sigma} = \boldsymbol{\tau} - p\delta \tag{2}$$

Constitutive law differs in subdomains $\Omega_1(t)$ and $\Omega_2(t)$. Consider that $\Omega_1(t)$ is occupied by the air and $\Omega_2(t)$ by SCC. The air can be considered as one-parameter (viscosity $\mu$) Newtonian fluid.

$$\boldsymbol{\tau} = \mu\boldsymbol{D} \tag{3}$$

However, fresh concrete flow has to be described by at least two parameters. The first one is yield stress $\tau_0$ which represents the initial resistance of the fresh concrete suspension to the flow. The second parameter, plastic viscosity $\mu_{pl}$, governs the flow. The natural choice is then the Bingham model, see [9]. Despite its simplicity, practical simulations have proved, that it is a suitable choice for describing fresh concrete behavior. The motion of Bingham fluid is governed by following equation

$$
\begin{cases}
\boldsymbol{\tau} = \left[\mu_{pl} + \frac{\tau_0}{\sqrt{J_2^e}}\right]\boldsymbol{D} & ; |\boldsymbol{J_2}| \geqslant \tau_0,\\
\boldsymbol{D} = \boldsymbol{0} & ; |\boldsymbol{J_2}| < \tau_0,
\end{cases}
\tag{4}
$$

where $\boldsymbol{D}$ denotes strain rate tensor is defined as a symmetric part of velocity gradient

$$\boldsymbol{D} = \frac{1}{2}\left(\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T\right). \tag{5}$$

The second invariants of deviatoric strain tensor $\boldsymbol{J_2^e}$ and the deviatoric stress tensor $\boldsymbol{J_2}$ are defined as

$$\boldsymbol{J_2^e} = \frac{1}{2}\boldsymbol{D} : \ \boldsymbol{D}, \tag{6}$$

and

$$\boldsymbol{J_2} = \frac{1}{2}\boldsymbol{\tau} : \ \boldsymbol{\tau}. \tag{7}$$

### 2.1. Numerical scheme

Following the usual finite element procedure and defining suitable finite-dimensional subspaces $\mathcal{S}^h \subset \mathcal{S}$, $\mathcal{V}^h \subset \mathcal{V}$ and $\mathcal{Q}^h \subset \mathcal{Q}$, the discretized problem states, see [10]: find $\boldsymbol{v^h} \in \mathcal{S}^h$ and $p^h \in \mathcal{Q}^h$ such that $\forall \boldsymbol{w^h} \in \mathcal{V}^h,\ \forall q^h \in \mathcal{Q}^h$:

$$
\begin{aligned}
&\int_\Omega \rho_j \boldsymbol{w^h}\frac{\partial \boldsymbol{v^h}}{\partial t}\mathrm{d}\boldsymbol{x} + \int_\Omega \rho_j\boldsymbol{w^h}\cdot(\boldsymbol{v^h}\cdot\nabla\boldsymbol{v^h})\mathrm{d}\boldsymbol{x} + \int_\Omega \nabla\boldsymbol{w^h} : \ \tau(\boldsymbol{v^h})\mathrm{d}\boldsymbol{x}\\
&\quad - \int_\Omega \nabla\cdot\boldsymbol{w^h}p^h\mathrm{d}\boldsymbol{x} - \int_\Omega \boldsymbol{w^h}\cdot\boldsymbol{b}\mathrm{d}\boldsymbol{x} - \int_{\partial\Omega}\boldsymbol{w^h}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{n}\mathrm{d}s + \int_\Omega q^h\nabla\cdot\boldsymbol{v^h}\mathrm{d}\boldsymbol{x}\\
&\quad + \sum_{el}\left[\int_{\Omega_e}\tau_{SUPG}(\boldsymbol{v^h}\cdot\nabla\boldsymbol{w^h})\cdot\mathcal{R}(\boldsymbol{v^h},p^h))\mathrm{d}\boldsymbol{x}\right]\\
&\quad + \sum_{el}\left[\int_{\Omega_e}\tau_{PSPG}\frac{1}{\rho}\nabla q^h\cdot\mathcal{R}(\boldsymbol{v^h},p^h)\mathrm{d}\boldsymbol{x}\right] = 0,
\end{aligned}
\tag{8}
$$

where

$$\mathcal{R}(\boldsymbol{v^h},p^h) = \left(\rho\frac{\partial\boldsymbol{v^h}}{\partial t} + \rho\boldsymbol{w^h}\cdot(\boldsymbol{v^h}\cdot\nabla\boldsymbol{v^h}) - \nabla\cdot\tau(\boldsymbol{v^h}) + \nabla p^h - \boldsymbol{b}\right)$$

represents the residuum of the momentum balance equation. The terms on the first two lines follow from the standard Galerkin discretization, the third line represents Streamline Upwind/Petrov-Galekin (SUPG) stabilization term due to convection effects and the fourth line provides Pressure Stabilizing/Petrov-Galekin (PSPG) stabilization for elements not satisfying Babuška-Brezzi condition. Note that PSPG terms are localized to the positions where zero sub-matrix appears in the standard Galerkin formulation, providing unique solvability of the matrix problem. The choice of stabilization parameters $\tau_{SUPG}$, $\tau_{PSPG}$ is a non-trivial task, and the details can be found in [5,10].

Semi-discretized formulation (see Eq. (8)) represents set of ordinary differential equations in time, which can be discretized using generalized mid-point rule. Since the solution procedure is not in the center of attention of this paper, we refer an interested reader to [11] for the details on the solution procedure.

## 3. Description of the fibers

In this section, description of the fiber orientation will be given. After the formulation of basic equations of a motion for a single fiber, probabilistic approach to the problem will be presented,