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The structure of dynamic correlations in multivariate stochastic volatility models[☆]

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ABSTRACT

This paper proposes two types of stochastic correlation structures for Multivariate Stochastic Volatility (MSV) models, namely the constant correlation (CC) MSV and dynamic correlation (DC) MSV models, from which the stochastic covariance structures can easily be obtained. Both structures can be used for purposes of determining optimal portfolio and risk management strategies through the use of correlation matrices, and for calculating Value-at-Risk (VaR) forecasts and optimal capital charges under the Basel Accord through the use of covariance matrices. A technique is developed to estimate the DC MSV model using the Markov Chain Monte Carlo (MCMC) procedure, and simulated data show that the estimation method works well. Various multivariate conditional volatility and MSV models are compared via simulation, including an evaluation of alternative VaR estimators. The DC MSV model is also estimated using three sets of empirical data, namely Nikkei 225 Index, Hang Seng Index and Straits Times Index returns, and significant dynamic correlations are found. The Dynamic Conditional Correlation (DCC) model is also estimated, and is found to be far less sensitive to the covariation in the shocks to the indexes. The correlation process for the DCC model also appears to have a unit root, and hence constant conditional correlations in the long run. In contrast, the estimates arising from the DC MSV model indicate that the dynamic correlation process is stationary.

1. Introduction

Static and dynamic covariance and correlation structures are used routinely for optimal portfolio choice, risk management, obtaining Value-at-Risk (VaR) forecasts, and determining optimal capital charges under the Basel Accord. Although the conditional volatility literature has examined the theoretical development of alternative dynamic covariance and correlation structures, this issue does not yet seem to have been examined in detail in the Multivariate Stochastic Volatility (MSV) literature.

For multivariate GARCH models, the most general expression is called the 'vec' model (see Engle and Kroner (1995)). The vec model parameterizes the vector of the conditional covariance matrix of the returns vector, which is determined by its lags and the vector of outer products of the lagged returns vector. A serious issue with the vec model is that it has many parameters to be estimated, and will not guarantee positive definiteness of the conditional covariance matrix without further restrictions. Bollerslev et al. (1988) and Ding and Engle (2001) suggested the diagonal GARCH model, which restricts the off-diagonal elements of the parameter matrices to be zero, and also reduces the number of parameters drastically in computing the conditional covariance matrix. Engle and Kroner (1995) proposed the Baba, Engle, Kraft and Kroner (BEKK) specification that guarantees the positive definiteness of the conditional covariance matrix, which is essential for obtaining sensible VaR forecasts.

In the context of modelling conditional correlations rather than conditional covariances, Bollerslev (1990) proposed the Constant Conditional Correlation (CCC) model, where the time-varying covariances are proportional to the conditional standard deviation derived from univariate GARCH processes. This specification also guarantees the positive definiteness of the conditional covariance matrix. Ling and McAleer (2003) develop the asymptotic theory for several constant correlation vector ARMA–GARCH models. As an extension of the CCC model, Engle (2002) suggested the Dynamic Conditional Correlation (DCC) model, which allows the conditional correlation matrix to vary parsimoniously over time.

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Table 1
Constant and dynamic correlation multivariate GARCH models.

Model	Specification	Number of parameters	Type
CCC: Bollerslev (1990)	$P_t = P$, $\omega_{ii,t} = w_i + \alpha_i \varepsilon^2_{ii,t-1} + \beta_i \omega_{ii,t-1}$	$\frac{m^2+5m}{2}$	C
DCC: Engle (2002)	$P_t = Q_t^{*-1} Q_t Q_t^{*-1}$, $Q_t = (1-\gamma-\delta)\, S + \gamma z_{t-1} z'_{t-1} + \delta Q_{t-1}$, $Q_t^* = \text{diag}\left\{\sqrt{q_{11,t}}, \ldots \sqrt{q_{mm,t}}\right\}$, $z_t = D_t^{-1}\varepsilon_t$, $\omega_{ii,t} = w_i + \alpha_i \varepsilon^2_{ii,t-1} + \beta_i \omega_{ii,t-1}$	$\frac{m^2+5m}{2} + 2$	D
BEKK: Engle and Kroner (1995)	$\Omega_t = C'C + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + B\Omega_{t-1}B'$	$\frac{5m^2+m}{2}$	D
Diagonal GARCH: Ding and Engle (2001)	$\Omega_t = C + A \circ \varepsilon_{t-1}\varepsilon'_{t-1} + B \circ \Omega_{t-1}$	$\frac{3m^2+3m}{2}$	D

In the column named 'Type', 'D' denotes dynamic conditional correlation models and 'C' denotes the constant conditional correlation model.

The development of dynamic correlation and covariance models has proceeded at a faster pace in the conditional volatility literature than in its stochastic volatility counterpart. Two reasons for this would seem to be the development of parsimonious multivariate dynamic conditional correlation models and their relative ease in estimation. McAleer (2005) provides a comprehensive comparison of a wide range of univariate and multivariate, conditional and stochastic, financial volatility models. Asai et al. (2006) discuss recent theoretical developments in the MSV literature.

Recently, Yu and Meyer (2006) developed the time-varying correlation model for the bivariate SV model, based on the Fisher transformation, as suggested by Tsay (2002) in a bivariate GARCH framework. Yu and Meyer (2006) also compared the empirical performance of nine alternative MSV models for a bivariate exchange rate series and found that MSV models that allow for time-varying correlations generally fit the data better. An obvious drawback of their analysis is the difficulty in generalizing their dynamic correlation model to a higher dimension. Yu and Meyer (2006, p. 366), note that "it is not easy to generalize the model into higher dimensional situations". The dynamic correlation MSV models that are developed in this paper are not restricted to be bivariate.

As a contribution to the development of parsimonious dynamic correlation MSV models that can be estimated with relative ease, Section 2 proposes two types of stochastic correlation structures for MSV models, namely the constant correlation (CC) MSV and dynamic correlation (DC) MSV models. The dynamic stochastic covariance matrices may be obtained easily from the dynamic stochastic correlation matrices. Alternative DC MSV models are developed. Both structures can be used for purposes of determining optimal portfolio and risk management strategies through the use of dynamic correlations, and for calculating Value-at-Risk (VaR) forecasts and optimal capital charges under the Basel Accord through the use of dynamic covariances. A technique is developed in Section 3 for estimating the DC MSV model using the Markov Chain Monte Carlo (MCMC) procedure. The properties of the estimation method are examined using simulated data, and various multivariate conditional volatility and MSV models are compared via simulation, including an evaluation of alternative VaR estimators. Section 4 provides an empirical example in which the model is estimated using three sets of empirical data. Some concluding remarks are given in Section 5.

2. Dynamic correlation models

In this section, the following definitions are used. Let ε_t be an m-dimensional stochastic vector. The operator vecd(.) creates a vector from the diagonal elements of a matrix. The operator $\circ$ denotes the Hadamard (or element-by-element) product. Let exp(.) denote the element-by-element exponential operator, and $\text{diag}\{x\} = \text{diag}\{x_1, \ldots, x_m\}$ denote the $m \times m$ diagonal matrix, with diagonal elements given by $x = (x_1, \ldots, x_m)'$. For any $m \times m$ matrix A, the matrix exponential transformation is defined by the power series expansion:

$$\text{Exp}(A) = \sum_{s=0}^{\infty} (1/s!) A^S,$$

where A^0 reduces to the $m \times m$ identity matrix and A^s denotes the standard matrix multiplication of A s times. Thus, in general, the elements of $\text{Exp}(A)$ do not typically exponentiate the elements of A.

2.1. Multivariate conditional volatility models

In the framework of the conditional volatility model, it is assumed that $E\left(\varepsilon_t \mid \Im_{t-1}\right) = 0$ and $E\left(\varepsilon_t \varepsilon_t' \mid \Im_{t-1}\right) = \Omega_t$, where $\Im_t$ is an information set up to period t. Thus, $\Omega_t = \left\{\omega_{ij,t}\right\}$ is the covariance matrix of ε_t conditional on past information. Let $D_t = \text{diag}\left\{\sqrt{\omega_{11,t}}, \ldots \sqrt{\omega_{mm,t}}\right\}$, so that the dynamic conditional correlation matrix, P_t, is defined by

$$P_t = D_t^{-1} \Omega_t D_t^{-1}. \quad (1)$$

The dynamic conditional covariance matrix, Ω_t, can be obtained from (1) by pre- and post-multiplication of both sides by the diagonal matrix to yield $\Omega_t = D_t P_t D_t$.

Table 1 shows the constant conditional correlation model and three dynamic conditional correlation models, namely, the CCC model of Bollerslev (1990), the DCC model of Engle (2002), the BEKK model of Engle and Kroner (1995), and the diagonal GARCH model of Ding and Engle (2001). The first two models are based on the specification of the conditional correlation matrix, P_t, while the remaining two models are based on the conditional covariance matrix, Ω_t. For the BEKK and diagonal GARCH models, the conditional correlation matrix defined by is dynamic. The CCC and DCC models are parsimonious, whereas the BEKK and diagonal GARCH models are not. The latter two models can be made more parsimonious by the imposition of suitable parametric restrictions. In multivariate conditional volatility models, the number of parameters increases to the order of m^2. When $m = 5$ (10), the numbers of parameters in the CCC and DCC models are 25 (75) and 27 (77), respectively, while those in the BEKK and diagonal GARCH models are 65 (255) and 45 (165),respectively.

Given the above, the primary features of the DCC model are that (i) each ε_{it} follows the univariate GARCH model, as in the estimation of the CCC model, which essentially follows a multiple univariate structure, and (ii) dynamic conditional correlations can be obtained through the addition of only two parameters to the CCC model. Thus, the DCC model is parsimonious in capturing dynamic correlations and covariances.

2.2. Multivariate stochastic volatility models

For MSV models, it is assumed that $E\left(\varepsilon_t \mid \Omega_t\right) = 0$ and $E\left(\varepsilon_t \varepsilon_t' \mid \Omega_t\right) = \Omega_t$, where the covariance matrix, Ω_t, is stochastic and symmetric positive definite. The first MSV model, which will