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Nonlinear identification of structures using ambient vibration data

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A B S T R A C T

Most of the practical engineering structures exhibit a certain degree of nonlinearity due to nonlinear dynamic characteristics of structural joints, nonlinear boundary conditions and nonlinear material properties. In this paper, we present a new technique based on null subspace analysis to detect the presence of nonlinearity and estimate the degree of nonlinearity in structures using ambient vibration data. Effectiveness of the proposed approach is demonstrated using carefully designed numerical examples and also using the experimental data. Extended version of the technique is presented to detect the nonlinearity using a single sensor data.

1. Introduction

Structural systems are often referred to as being linear or nonlinear. However, all real structures are inherently nonlinear. Nonlinear behavior is observed even in rather simple structures like plates and beams, as a result of buckling or large deformation related effects. The nonlinear behavior of a structure may be also possible due to a local (friction, joint and link flexibility, backlash and clearance, nonlinear contact) or a global (geometric nonlinearities, nonlinear material behavior) nonlinearities. However, in most of the cases they are usually approximated by a linear model for the purpose of dynamic analysis and design, solely due to computational ease and convenience. For linear models, modal based methods are most widely used in structural system identification, model updating, structural control and health monitoring.

However, the presence of nonlinearity in a structural system changes its behavior, thus making the use of the linear model improper and in the majority of cases even impossible. The basic principles that apply for a linear system, which form the basis for modal analysis are not valid anymore for nonlinear systems. The superposition and the homogeneity as well as the Maxwell reciprocity principles do not apply for a nonlinear system. A nonlinear mechanical system shows a tendency to redistribute the energy of the input spectrum. This results in modulation, super and sub-harmonics and broadband spectra in some areas. The generation of harmonics depends on the excitation. The frequency response functions are also excitation dependent, which makes impossible their further application for modal analysis. Modal models are unsuitable to predict the behavior of nonlinear systems. Accordingly, new tools for detection, quantification and modeling of nonlinearities in dynamical systems are necessary. Several methods have recently been developed for detecting the presence of nonlinearity in a system [1–6]. Some procedures rely on characteristic features for nonlinear systems, like the distortion of the FRF and Hilbert transform of FRF plots. During recent years, considerable research work has been reported on modeling nonlinear behavior of dynamic systems, suggesting a variety of approaches. The use of Volterra series [7] to describe the nonlinear systems is one of the most widely accepted one.

Several methods exist in the literature for detecting the presence of non-linearity. These methods can be broadly classified as frequency domain and time domain analysis based techniques [1]. The frequency domain based nonlinear detection methods include homogeneity test and Hilbert transform of frequency response functions, coherence function, Hilbert marginal spectrum, wavelet packet transform component correlation coefficient, bispectral analysis, wavelet packet energy spectrum, etc. Similarly, the time domain nonlinear indicators include instantaneous frequency, holder exponent, auto and cross correlations of time history response, etc. More details on these detection methods can be found in Kerschen et al. [1], Worden and Tomlinson [3] and Hickey and Worden [8].

Civil structures will inevitably suffer a certain level of deterioration during its service life owing to corrosion and/or fatigue

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damage, aging of construction materials, long-term effect of loads, sudden attacks of accidental and natural catastrophes. Damage will make the structural properties, nonlinear and vary with time. Thus the measured structural vibrations are often time-varying and nonstationary [9]. Moreover, nonlinearities exist in all civil structures to some degree, which arise from the nonlinearity of material, changing of geometric configurations when a structure experiences large deformations (i.e., geometric nonlinearity) or changing of boundary conditions and system connectivity (e.g. a roller support is either in contact or not in contact, cracks opening and closing). Therefore, structural vibration properties (frequencies, mode shapes and damping) vary with time. The identification process requires to analyze the nonlinear and non-stationary structural vibration data. Moreover, for civil structures, it is often more convenient to measure the ambient vibration data than forced response data. Hence we emphasize more on methods which enables one to use the ambient data for detection of nonlinearity. Most of the techniques listed above require both input and output measurements to detect the presence of nonlinearity. Even though there are few techniques, which use only ambient vibration data, they are highly susceptible to measurement noise. Recently, Hot and Kerschen [10] has proposed a technique based on principal component analysis for identifying the presence and the level of nonlinearity present in the structures, where we can conveniently utilize ambient vibration data.

In this paper, we present the application of a technique based on null subspace for detection of the presence and also the degree of nonlinearity in civil engineering structures. Several numerical examples are solved to demonstrate the effectiveness of the proposed approach for detection of nonlinearity, which includes few numerically simulated examples and experimental results of the benchmark structures. The numerical investigations carried out in this paper clearly indicate that the proposed null subspace method is an effective tool for detecting the presence of nonlinearity even in the presence of measurement noise. Studies also reveal that we can precisely assess the degree of nonlinearity of the structure using this method. In this paper, we will draw comparisons with the existing PCA [11] based detection technique and highlight the higher degree of sensitivity and also versatility of the proposed technique.

2. Null space based approach

The null space based algorithm is partially inspired by the concept of subspace identification. However, for detection of the presence of nonlinearity, we do not require modal identification. The null space method has earlier been applied to damage detection [12]. In this paper, we have proposed to explore this method to detect the presence of nonlinearity and also to assess its severity.

In order to illustrate the null space based approach for detecting the presence of nonlinearity, we consider a structural system for which the time history response is to be measured. Since, the response is usually measured in the form of acceleration time history, the structural system is instrumented with 'm' accelerometers. The acceleration time history response is measured periodically resulting in several data sets. The sampled data can be partitioned into several data subsets, in case of continuous online monitoring of the structure. We can construct a block Hankel matrix $\mathbf{H}_{p,q}$ consist of 'p' block rows and 'q' block columns of the output covariance matrix for each data subset and it can be written as

$$\mathbf{H}_{p,q} = \begin{bmatrix} \Lambda_0 & \Lambda_1 & \cdots & \cdots & \Lambda_{q-1} \\ \Lambda_1 & \Lambda_2 & \cdots & \cdots & \Lambda_q \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \Lambda_{p-1} & \Lambda_p & \cdots & \cdots & \Lambda_{p+q-2} \end{bmatrix}; \quad q \geqslant p \tag{1a}$$

The indices p and q in the Hankel matrix define the number of considered time shifts and should be chosen based on the assumed system order n, i.e. $q = n - p + 1$. In the present work, we have considered $p = q$. Λ_i represents the output covariance, which is estimated from a set of N output data samples of the acceleration time history response.

$$\Lambda_i \simeq \frac{1}{(N-i)} \sum_{k=1}^{N-i} y_{k+i} y_k^T; \quad 0 \leqslant i \leqslant N-1 \tag{1b}$$

where $\{y\}$ is the acceleration time history response of a particular sensor. y_k refers to the acceleration at kth time step. It may be pointed out that the block Hankel matrices give an instantaneous representation of responses and therefore these block Hankel matrices improve the sensitivity of the detection of nonlinearity in the structure when compared to the observation matrices. Hence we prefer to use the block Hankel matrices in the present work.

According to the subspace identification theory [13,14], the state matrices can be extracted from the Hankel matrix which represents the modal subspace spanned by the mode-shape vectors of the structure. We can subsequently identify the modal parameters (natural frequencies, damping ratios and mode shapes) also. However, it should be mentioned here that our aim is not in identifying the precise values of modal parameters or other structural features. Instead, we are more concerned with only relative changes of the characteristic features that are necessary to indicate the presence of nonlinearity. For this purpose, a method based on the null subspace concept of these Hankel matrices is used.

Null Subspace analysis is basically a principal component analysis of the Hankel autocorrelation matrix. Performing the singular-value decomposition (SVD) on the weighted Hankel matrix, we get:

$$\bar{\mathbf{H}} = \mathbf{W}_1 \mathbf{H}_{p,q} \mathbf{W}_2 \approx [\mathbf{U}_1 \quad \mathbf{U}_2] \begin{bmatrix} \mathbf{S}_1 & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{V}_1 \quad \mathbf{V}_2]^T = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1^T \tag{2}$$

where $\mathbf{W}_1$ and $\mathbf{W}_2$ are weighting matrices. Since we are only interested in monitoring relative changes of the characteristic features that are necessary for detection of the presence of nonlinearity in the present work and not on the modal parameter estimation, the invertible weighting matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ are chosen as identity matrices for simplicity and the same was also suggested by Yan and Golinval [12]. However, a discussion on the effect of the weighting matrices on modal identification can be found in Van Overschee and De Moor [13].

The diagonal matrix $\mathbf{S}_1$ contains $n = 2N_m$ non-zero singular values sorted in decreasing order, where N_m indicates the number of modes. The following four fundamental subspaces can be extracted by performing SVD on the weighted Hankel matrix, $\bar{\mathbf{H}} \in \Re^{r \times c}$ where $r = m \times p$ and $c = m \times q$,

i. $\mathbf{U}_1$ with maximum number (n) of independent column vectors that span the column space of $\bar{\mathbf{H}}$.
ii. $\mathbf{V}_1^T$ with maximum number (n) of independent row vectors that span the row space of $\bar{\mathbf{H}}$.
iii. $\mathbf{U}_2$ with maximum number $(c - n)$ of independent column vectors that span the column null-space of $\bar{\mathbf{H}}$.
iv. $\mathbf{V}_2^T$ with maximum number $(r - n)$ of independent row vectors that span the row null-space of $\bar{\mathbf{H}}$.

It may be pointed out that the exact order n of the system is not fixed and it varies depending on the input signal. In order to find the exact order n, we have to scan through the singular values in $\mathbf{S}$, till the values are equal to zero or very insignificant and take the left hand side vectors, $\mathbf{U}_1$ corresponding to those null singular values.