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A linear complementarity method for dynamic analysis of bridges under moving vehicles considering separation and surface roughness

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ABSTRACT

In the present paper, a linear complementarity method for a vehicle-bridge dynamic system considering separation and random roughness is established. By introducing the linear complementarity relationship between the relative displacement of the wheels and the bridge at the contact points, the dynamic interaction problem of the vehicle-bridge coupled system is transformed into a standard linear complementarity problem, and two models with different connection relations between the wheels and the bridge are proposed. The presented models characterize the system with one unified formulation whether the wheels separate from the bridge or not, and the conventional trial-and-error iterative process in numerical simulation is avoided. In the numerical examples, the proposed method is verified by comparing it with the conventional method, and it is found that the velocity, the vehicle to bridge mass ratio and the road roughness have a significant influence on separation. By considering a vehicle model of three rigid bodies with four wheels and the randomness of the rail roughness in train–track–bridge system, the possibility of separation and the expectation of the maximum separation distance at different velocities are studied. The results show that it is very useful to carry out a stochastic analysis of the system and consider the influence of separation in vehicle and bridge design.

1. Introduction

The dynamic behaviour of beam structures, such as railway bridges, subject to moving loads has been investigated for over a century since Stokes [1] firstly brought this problem into attention. It is of great interest in many engineering applications, such as the design of bridges, railway tracks and cableways, and a large number of papers related to this problem have been published to predict the dynamic responses of simple supporting structures under moving loads [2–8]. The early structural engineers found that under moving loads, structural dynamic deformations and stresses can be significantly higher than those caused by corresponding static loads [2,3]. Frýba [4] described the basic postulation of moving load problems and their analytical solutions. Olsson [5] discussed the assumption inherent in the moving force problem and solved it by the finite element method. In the moving force problem, the inertial force of the moving structure was neglected which would possibly miss some aspects of the physics involved and could not capture the interaction behaviour between the moving structure and the bridge during the travelling [7]. The moving mass problem was then suggested, which brings some improvements to the moving force model. The moving structure was considered a single mass [9,10] first, and an oscillator model was then considered which would be more appropriate for some applications [11–13]. More models with different degrees of complexity are used to represent realistic vehicles [14–16].

As the position of the moving structure changes with time, the coefficient matrices of the moving mass problem are time-dependent. There are two conventional ways to simulate the time-dependent equations of motion of vehicles-bridge systems. The first way assumes that the vehicle is in sliding contact with the bridge and solves the coupled equations of motion for the whole vehicle-bridge system through numerical integration in time domain, which requires a very small time step [17]. The other way is based on the uncoupled iteration method, in which each system (both the vehicles and the bridge) is solved separately and an iterative process in each time step is performed to find the equilibrium between the bridge and vehicle wheels. By using a proper estimate of the interaction forces, the accurate solution of the system can be

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obtained for a larger time step [17,18]. The paper adopts the second way.

All these papers mentioned above took no consideration of the possibility of separation between the moving object and the bridge, although separation was shown to be possible in theory and studied in some numerical simulations [19–25]. Lee [20] is perhaps the first researcher to study the separation between the moving mass and the supporting structure. In that case, the contact force was time-dependent and became zero under certain circumstances. The transition of the moving contact force from positive to zero was considered to be the onset of separation between the moving and supporting structures. It is found that the most important parameters influencing the separation are the sliding speed and the mass ratio between the moving mass and the beam [21]. Stăncioiu et al. [22,23] and Baeza and Ouyang [24] studied separation and reattachment of an oscillator moving along a beam structure, and put forward a simplified method for computing the dynamic responses after the impact at reattachment. On the other hand, the road roughness of the bridge deck is a real physical phenomenon which almost exists in all bridges. It is found that road roughness magnifies the dynamic response due to the moving action of the mass, which would possibly be detrimental to the safety and serviceability of structures [26–28]. Furthermore, for a railway bridge, the rail roughness is a random process, which would increase the possibility of separation and make the responses harder to predict. Cheng et al. [25] studied separation considering the surface roughness of the bridge modelled as a continuous beam, and proposed an algorithm to account for the impact on the reestablishment of contact. It is found that separation often occurs in the vicinity of the crests of roughness, and the velocity has a significant influence on separation.

Many researchers modelled the vehicle as an oscillator, which has only one wheel, thus only one equation of motion is required to demonstrate separation. But for a multi-wheel vehicle model, loss of contact for any wheel of the vehicle changes the time-dependent matrices by adding a DOF corresponding to the wheel separating from the bridge. Extra equations of motion are needed to describe all the possible combinations due to different wheels separating from the bridge, which makes it too complicated to deal with.

In this paper, a linear complementarity method [29,30] for vehicle-bridge system considering separation with road roughness is established. In the present simulation, a unified equation of motion is constructed to describe the system whether the wheels separate from the bridge or not. Therefore, the complex trial-and-error iterative processes in conventional numerical simulation are avoided (that is, the assumptions or the iterative process for determination of contact states is not required in the proposed method). The moving oscillator model with separation is firstly used to verify the present method by comparing it with the moving oscillator model with permanent contact. It is found that the road roughness significantly influences the separation region at various values of the velocity ratio and the vehicle to bridge mass ratio considered. The random track irregularity in railway and a multi-wheel vehicle are considered later. By implementing the Monte Carlo simulation, the possibility of separation and its influence on the dynamic responses are studied. Finally, the expectation of maximum distance of separation is determined, which is suggested as an indicator of the influence of separation.

2. Equations of motion of vehicle bridge systems and separation

As shown in Fig. 1, a simple beam model for a vehicle bridge is presented first. The vehicle is modelled as an oscillator with two degrees of freedom z_1 and z_2. The sprung mass and the wheel are connected by a spring-damper, with stiffness k_1 and damping c_1. The oscillator is moving along the bridge at a constant velocity V.

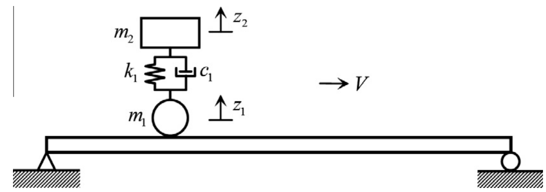

Fig. 1. A simple vehicle–bridge coupled system model.

The vertical displacement z_2 of the sprung mass m_2 and the vertical displacement z_1 of the wheel m_1 are governed by a set of two equations of motion

$$\begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix}\begin{Bmatrix} \ddot{z}_2 \\ \ddot{z}_1 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix}\begin{Bmatrix} \dot{z}_2 \\ \dot{z}_1 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix}\begin{Bmatrix} z_2 \\ z_1 \end{Bmatrix} = \begin{Bmatrix} -m_2 g \\ f_v - m_1 g \end{Bmatrix} \tag{1}$$

where the overdot stands for the total derivative d/dt and f_v is the interaction force at the wheel, g is gravitational acceleration. Eq. (1) can be written in matrix form

$$\boldsymbol{M}_v\ddot{\boldsymbol{z}}_v + \boldsymbol{C}_v\dot{\boldsymbol{z}}_v + \boldsymbol{K}_v\boldsymbol{z}_v = \boldsymbol{F}_v(t) \tag{2}$$

On the other hand, the equation of motion of a finite element model of bridge can be written as

$$\boldsymbol{M}_b\ddot{\boldsymbol{u}}_b + \boldsymbol{C}_b\dot{\boldsymbol{u}}_b + \boldsymbol{K}_b\boldsymbol{u}_b = \boldsymbol{F}_b(t) \tag{3}$$

where $\boldsymbol{M}_b$, $\boldsymbol{C}_b$ and $\boldsymbol{K}_b$ are the mass matrix, damping matrix and stiff matrix of the bridge respectively. $\boldsymbol{F}_b(t)$ is the load vector acting on the bridge, which can be obtained by

$$\boldsymbol{F}_b(t) = -\boldsymbol{R}(t)f_v(t) \tag{4}$$

where $\boldsymbol{R}$ is the influence matrix, which transforms the non-nodal load into equivalent nodal load, and can be written as

$$\boldsymbol{R}(t) = \boldsymbol{T}\boldsymbol{N}^{\mathrm{T}}(\xi) \tag{5}$$

in which $\boldsymbol{N}(\xi)$ is the shape function vector of the bridge element in contact with the wheel,

$$\boldsymbol{N}(\xi) = [N_1(\xi) \quad N_2(\xi) \quad N_3(\xi) \quad N_4(\xi)]^{\mathrm{T}} \tag{6}$$

where ξ is the local coordinate of the contact point related to the wheel's horizontal location which is changing with time, and

$$\begin{aligned} N_1 &= 1 - 3\xi^2 + 2\xi^3, \quad & N_2 &= l(\xi - 2\xi^2 + \xi^3) \\ N_3 &= 3\xi^2 - 2\xi^3, & N_4 &= l(\xi^3 - \xi^2) \end{aligned} \tag{7}$$

while l is the length of the contact element, $\boldsymbol{T}$ is the position transforming matrix of the element, consisting of 0 and 1. Similarly, the displacement u_c of the bridge-wheel contact point can be obtained by

$$u_c(t) = \boldsymbol{R}^{\mathrm{T}}(t)\boldsymbol{u}_b(t) \tag{8}$$

By applying the mode superposition method to the bridge, the displacement of the bridge can be expressed as

$$\boldsymbol{u}_b = \sum_{i=1}^{n}\boldsymbol{\varphi}_{bi}q_{bi} = \boldsymbol{\Phi}_b\boldsymbol{q}_b \tag{9}$$

where $\boldsymbol{\Phi}_b = [\boldsymbol{\varphi}_{b1} \quad \boldsymbol{\varphi}_{b2} \quad \cdots \quad \boldsymbol{\varphi}_{bn}]$ is the modal matrix of the bridge, and $\boldsymbol{q}_b = [q_{b1} \quad q_{b2} \quad \cdots \quad q_{bn}]^{\mathrm{T}}$ is the modal displacement vector for the bridge. Thus the equation of motion of the bridge can be written as

$$\overline{\boldsymbol{M}}_b\ddot{\boldsymbol{q}}_b + \overline{\boldsymbol{C}}_b\dot{\boldsymbol{q}}_b + \overline{\boldsymbol{K}}_b\boldsymbol{q}_b = \overline{\boldsymbol{F}}_b \tag{10}$$