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Multi-scale modelling for two-dimensional periodic structures using a combined mode/wave based approach

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ABSTRACT

In this article, an efficient numerical approach is proposed to study the vibration of two-dimensional periodic structures. The method combines the advantages of mode-based Component Mode Synthesis (CMS) and wave-based Wave Finite Element Method (WFEM). It begins with a modal description of a mesoscopic unit cell using CMS. Subsequently, WFEM is applied to the macroscopic structure, which is considered as a waveguide. It exploits fully the periodic propriety of the structure since only one unit cell needs to be modelled. The introduction of CMS is able to reveal the influence of local dynamics of unit cell on the global behaviour of the structure, and speed up the computation of eigen-problem. The wave-mode duality is discussed which assures the combination of the two methods. The effectiveness of the proposed approach is illustrated via an example of two-dimensional beam grid. The convergence criteria of the model reduction is given and the equivalence of cell modes and stationary waves at bounding frequencies of stop bands is verified.

1. Introduction

Two-dimensional (2D) periodic structures that can be obtained by repeating a single unit cell are widely employed in various engineering domains. Their applications spread from the sandwich panels, stiffened plates, truss beams used in aerospace and marine structures, to the perforated plate used in the tube sheet heat exchangers in nuclear power plants. The wide application and rich dynamic behaviours of periodic structures have attracted lots of researchers for several decades [1–5].

Numerous wave-based methods have been developed during the study or design of homogeneous or periodic structure. The Semi-Analytical Finite Element (SAFE) and Wave Finite Element Method (WFEM) are two efficient tools to study these structures. In the SAFE method, the displacement field is formulated following decomposition into plane waves in the direction of propagation, and using finite elements in the direction perpendicular to propagation one. The numerical method – WFEM has been developed to overcome the limitations of analytical model in SAFE by combining the periodic structures theory with FEM. It has been extensively applied to study the propagation in homogeneous or periodic waveguides in last decades [3,6–8]. The method is based on periodic structures theory [9,10], converting the study of the whole periodic structure into a single unit cell. Free harmonic wave motion can be deduced from the dynamic stiffness matrix of the unit cell. The obtained wave motion corresponds to the wave basis. All waves propagating (free or forced) in this structure can be decomposed into this wave basis. However, for waveguides with complex cross-section, computation based on wave basis may be time-consuming. So several reduction formulations of WFEM have been proposed. Droz et al. have proposed a reduction formulation to determine the propagating wave in 1D refined model of a laminated composite beam [11]. The main idea is to use the wave basis at cut-on frequencies to describe the wave basis in the whole frequency range. Then the spectral problem on the dynamic stiffness matrix of unit cell is expressed by reduced wave coordinates. Similarly, a reduced wave basis expansion method has been proposed by Hussein for fast calculation of band structure in 2D periodic structures [12]. However, instead of forming the reduced wave basis using the waves basis at cut-on frequencies, the eigenvectors corresponding to a selected wavenumbers are employed. Mencik et al. have proposed a reduction technique by selecting the wave modes that are relevant for computing the forced response of elastic waveguides. However, the numerical costs for computation of the wave basis are not reduced [13]. Mead has presented a reduced method by identifying, in a preliminary investigation, the characteristic waves which contribute to the motion

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of the structure. Then the computation time to calculate the wave basis as well as the forced response of the structure is reduced [14].

The aforementioned reduction techniques are interesting to study waveguides with complex cross-section with a large number of coupling coordinates. But few reduction method is developed to deal with the periodic structures with numerous internal DOFs in the unit cell, which may also lead to excessive computational time. In this paper, a mode-based a priori reduction method is coupled with WFEM. The reduced model of the unit cell is computed based on Craig–Bampton method, the fixed boundary Component Mode Synthesis (CMS) method [15]. The numerous physical internal coordinates of the unit cell are represented by a reduced set of modal coordinates, while the physical coupling coordinates are conserved. The reduction is robust and easy to implement to WFEM formulation. The reduced model is valid in all the frequency range of interest, while the aforementioned wave-based reductions depend on the band of frequency studied. In addition, the selection of the modal basis allows us to study the influence of local dynamics on the global behaviours. The proposed method combines the modal description of vibration on unit cell scale and wave description on the whole structure. The equivalence of two descriptions is known as "wave-mode duality" [16]. More details about the advantage of this combination can be found in Section 2.

As for periodic structures, an interesting dynamic behaviour is stop bands (called also band gaps) and directivity of propagation. A lot of work has been done in the study of stop bands, for example to maximise stop bands or to localise stop bands in certain frequency regions [17–19]. The proposed method can be used to predict the stop bands, and also to study the wave shapes at the bounding frequencies of the stop bands. According to Mead [20], the bounding frequencies of the stop bands correspond to natural frequencies of the unit cell under appropriate boundary conditions. On the numerical example of a beam grid, the bounding frequencies and associated wave shapes are compared with natural frequencies and modes shapes.

In this work, general formulations for modelling periodic structures using a combined mode/wave approach are proposed. On the whole, the paper focuses on two main objectives:

1. Offering efficient numerical tools for the predictions of wave propagation characteristics, to study the influences of local dynamics on global behaviours, and minimise computational cost.
2. Discussing some aspect of the wave-mode duality of the structure vibration. Analysing equivalence of the cell modes and stationary wave at bounding frequencies of stop band.

The paper is organised as follows: in Section 2, the modal description and wave description of vibration are explained briefly. The terminology concerning wave description is defined to avoid ambiguity. The advantages of the proposed approach are described. In Section 3, the proposed method is developed in detail. Different representations of the results are explained as well in this section. Subsequently, a numerical example of a beam grid is fully presented in Section 4. The effectiveness of the proposed method is illustrated and the wave beaming effect is studied. The wave-mode duality at bounding frequencies of stop band is discussed. Finally, conclusions and perspectives of this paper are presented in Section 5.

2. Terminology: wave-mode duality, macroscopic and mesoscopic scales

It is well known that the response of a vibrating system can be viewed either in terms of modes or in terms of elastic wave motion, known as "wave-mode duality". Plenty of discussions regarding the relationship between the wave and the modal descriptions can be found [16,21]. For 1D waveguide, the wave-mode duality is demonstrated in a mathematical sense. The vibration can be written both as a sum of right- and left-going components and a sum of participation of all the modes under the same boundary conditions. However for 2D structure, the duality has not been proved precisely due to ill-conditioning problem in numerical computation [16].

The structure studied in this paper is called "two-dimensional periodic", which means that it is periodic in two directions, for example x and y direction. The property can vary in the 3rd direction, the z direction. An illustration of two-dimensional periodic structure is given in Fig. 1. Such structure could be a stiffened plate in fuselage or a honeycomb sandwich. Different scales related to multi-scale modelling of 2D periodic structure in this paper is defined as follows:

Macroscopic scale: The whole periodic structure, as shown in Fig. 1a. It can be infinite or finite with a large number of unit cells.

Mesoscopic scale: The unit cell of the structure, as shown in Fig. 1b. By repeating this unit cell in x and y directions, one can obtain the whole periodic structure.

In this paper, the proposed method employs the modal description of vibration on the mesoscopic scale of unit cell, and the wave description is used on the macroscopic scale of periodic structure.

2.1. Analysis of periodic structure using modal approach – Component Mode Synthesis

Component Mode Synthesis (CMS) is an efficient mode-based method to study periodic structures [15]. CMS enables structures to be analysed as a set of components, which form the whole structure when joined together. It has many advantages, such as allowing analysis to proceed independently on each component, and making each analysis smaller. One of the most common methods is Craig–Bampton method. To build the component (unit cell) model, we use a reduced basis of fixed boundary modes $\mathbf{\Psi}_{\mathrm{C}}$. Constraint static modes of the component interfaces $\mathbf{\Psi}_{\mathrm{bd}}$ are also introduced. The modal basis $\mathbf{\Psi}^{\mathrm{s}}$ of the structure can be deduced by assembling all the components, after expanding the reduced modal coordinates to the physical coordinates. Subsequently, the physical displacement $\mathbf{q}$ is related to the modal displacement η using the following equation:

$$\mathbf{q}(\vec{r},\omega) = \sum_{i=1}^{n}\eta_i(\omega,\omega_{0i})\boldsymbol{\psi}^{\mathrm{s}}_{\mathrm{i}}(\vec{r},\omega_{0i}) \quad (1)$$

where $\vec{r}$ represents the physical coordinates. "Modes shapes" $\boldsymbol{\psi}^{\mathrm{s}}_{\mathrm{i}}$ are associated with natural frequency ω_{0i}, which form a "structural modal basis" $\mathbf{\Psi}^{\mathrm{s}} = [\psi^{\mathrm{s}}_{1}, \psi^{\mathrm{s}}_{2} \ \ldots, \psi^{\mathrm{s}}_{\mathrm{n}}]$. In this paper, the modes (cell modes and structural modes) shapes are represented by $\boldsymbol{\psi}_{\mathrm{i}}$, whereas wave shapes are represented by $\boldsymbol{\phi}_{\mathrm{i}}$ in wave description.

2.2. Analysis of periodic structure using wave approach – Wave Finite Element Method

In the wave description, the displacement field (free or forced) of structure is regarded as a sum of harmonic waves:

$$\mathbf{q}(\vec{r},\omega) = \sum_{i=1}^{n}a_i(\omega,\vec{x_1},k_i)\phi_i(\omega,\vec{x_2},k_i) \quad (2)$$

where ω is the frequency of the propagating wave. Physical coordinates $\vec{r}$ are divided into $(\vec{x_1},\vec{x_2})$, with $\vec{x_1}$ representing coordinates in the propagation direction and $\vec{x_2}$ representing the local coordinates in the unit cell. Instead of the standard FEM where the whole