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Optimization of cross-section of actively bent grid shells with strength and geometric compatibility constraints

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ABSTRACT

The use of bending as self forming process allows the realization of shape-resistant systems, such as grid shell structures. Here, a numerical method for optimization of the cross-section of actively bent structures is introduced. For a given load distribution, the optimization objective consists of normalizing the bending stresses to a given value on the entire structure. In addition, strength and geometric compatibility constraints are taken into account. The method is demonstrated by numerical examples. Further, in order to handle the large displacements involved, a co-rotational Finite Element formulation is adopted and modified to take into account the changes in stiffness that occur in the forming process of active bending systems. The modified co-rotational formulation is solved for static equilibrium using a Dynamic Relaxation scheme, and is tested against the analytical solutions of some preliminary test cases, as well as experimental results, and shown to be 'accurate'.

1. Introduction

The term 'Active Bending' defines a category of structural systems in which bending is used as a self-forming process [1]. For instance, the realization of grid shell systems obtained by assembling an initially flat mat made of continuous elastic rods (e.g. fiber reinforced polymers [2] or timber [3–8] and successive forming by means of adjustable scaffolding or temporary crane-cable systems. Shell systems derive their strength and stiffness from their inherent doubly curved shape, allowing them to work mainly in membrane action under the effect of external loads. Nevertheless, a certain amount of out-of-plane stiffness is required to resist inextentional deformations [9].

The double-layer technique, first adopted in the design of the Mannheim timber grid shell for the Garden Festival [3] allows tighter curvatures to be obtained compared to a single-layer mat made from rods with equivalent cross-sectional area. Once the forming process is complete, sliding between overlapping laths is constrained by inserting shear blocks in between the laths making up the single rib (see Fig. 1) thus enhancing the out-of-plane bending stiffness of the equivalent continuous shell.

The shape of such (actively bent) grid shell systems can be modeled by performing a preliminary simulation of the forming process by means of non-linear finite element procedures. Thus the resulting geometry can then be used as a basis for further structural analyses. Nevertheless, the effect of residual pre-stress forces on the overall structural behavior, as well as the change in stiffness due to the presence of shear blocks, needs to be taken into account when assessing the actual load-carrying capacity of the structure.

A comprehensive numerical procedure is introduced here to solve the initial form finding phase, the construction process simulation and successive load calculations of such actively bent grid shell systems. A modified co-rotational beam element with six degrees of freedom (DoF), in conjunction with the Dynamic Relaxation method (DR), allows the change in stiffness of the post-formed mat to be taken into account whilst, maintaining the resulting equilibrium configuration of the double-layer mat with sliding connections. Consequently, an optimization method for deriving the double-layer cross-section is proposed. For a given load configuration, the iterative method allows the bending stress ratios to be 'consolidated', resulting in a grid shell geometry with members having variable cross-section. Practical issues, rising from the fact of having a different cross-section for each member, can be handled by post-rationalizing members into groups, or providing fabrication's methods that allow to 'accurately' reproduce the linear variation of each member's profile. Further discussion about this will be addressed in the conclusions with a prospective from the structural point of view.

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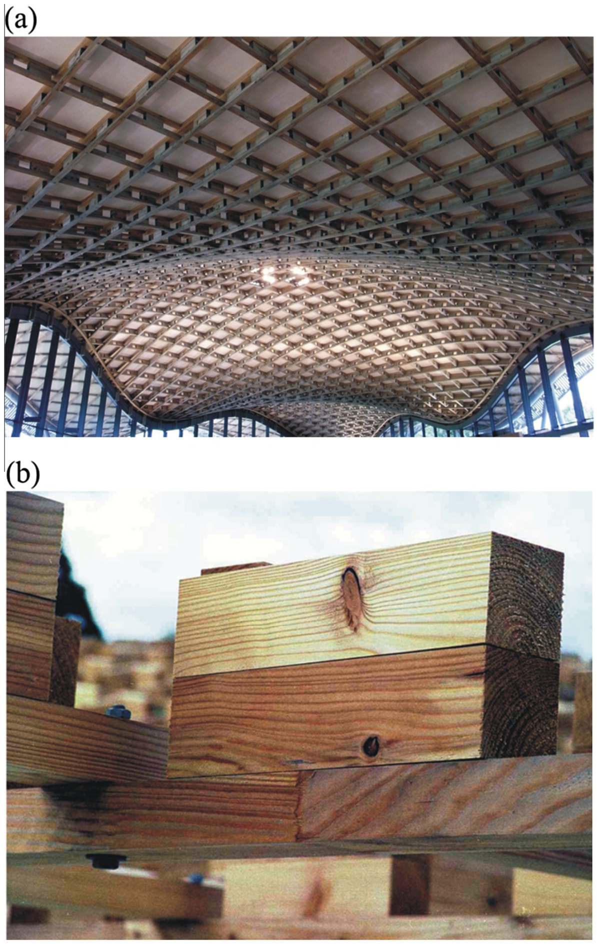

Fig. 1. Savill Garden grid shell, Windsor, UK 2006 [10]: (a) Internal view. (b) Detail of the shear block connection. (Photos courtesy – Richard Harris.)

2. Preliminary theory

2.1. Co-rotational formulation

In order to handle the large displacements and rotations involved in the form finding process of actively bent structures, a co-rotational formulation [11,12] for a three-dimensional beam element is adopted. Unlike the *Total Lagrangian* and *Updated Lagrangian* formulations [13], in the co-rotational approach the motion of the element is treated as a result of a rigid motion plus a deformation.

Assuming a geometry represented by a discrete set of nodes **P** with coordinate $\bar{p}_i$ with arbitrary initial position in the Cartesian coordinate system:

$$\mathbf{P} = \{\bar{p}_1 \ldots \bar{p}_i \ldots \bar{p}_{m^\circ}\}; \quad \bar{p}_i = [x\ y\ z] \tag{1}$$

and a connectivity list **E** storing the nodes' indices of the element ends $(1,2)$:

$$\mathbf{E} = \{\mathbf{e}_1 \ldots \mathbf{e}_j \ldots \mathbf{e}_{n^\circ}\}; \quad \mathbf{e}_j = \{i_1, i_2\} \tag{2}$$

the rigid motion of the jth element is determined by an *auxiliary vector* $\bar{p}_j$ connecting the element end nodes $(\bar{p}_{i_1}, \bar{p}_{i_2})$. Then, assuming a 'right-handed' local reference frame $\{\bar{x}_i, \bar{y}_i, \bar{z}_i\}$ for the generic i node, the element deformation (local rotational and axial displacements) is determined by computing the local frame orientation of end nodes with respect to the element vector position $\bar{p}$.

With reference to Fig. 2, the local rotations of the $\bar{e}_j$ element around the local ($\bar{x}_i$ and $\bar{y}_i$) axes at its start node i_1 are $\theta_{x,1}$ and $\theta_{y,1}$, while $\theta_{x,2}$ and $\theta_{y,2}$ are the rotations around (the local frame) at its i_2 end node. Whereas, φ is the angle of twist while e is the axial shortening/elongation. The local shear displacements are not explicitly set out because of the reference axes choice (at a nodal level instead of element level).

From the element's local rotations and displacements, the corresponding element's ends reactions can be obtained by differentiating the beam's expression of total strain energy U [14] thus obtaining the bending moments $M_{x,1}, M_{y,1}, M_{x,2}$ and $M_{y,2}$, the torsion moment M_φ and the axial force N. Again, the local shear forces are missing due to the reference axes choice. Such an element's local reactions are a function of its material and geometric stiffness i.e.: the second moments of area (I_x and I_y), torsional constant (J), cross-sectional area (A), element's unstressed length (L_0), Young's and shear moduli (E and G):

$$N = \frac{EA}{L_0}e; \quad M_\varphi = \frac{GJ}{L_0}\varphi \tag{3}$$

$$M_{x,1} = \frac{NL_0}{30}(4\theta_{x,1} - \theta_{x,2}) + \frac{2EI_x}{L_0}(2\theta_{x,1} + \theta_{x,2}) \tag{4}$$

$$M_{x,2} = \frac{NL_0}{30}(4\theta_{x,2} - \theta_{x,1}) + \frac{2EI_x}{L_0}(2\theta_{x,2} + \theta_{x,1}) \tag{5}$$

$$M_{y,1} = \frac{NL_0}{30}(4\theta_{y,1} - \theta_{y,2}) + \frac{2EI_y}{L_0}(2\theta_{y,1} + \theta_{y,2}) \tag{6}$$

$$M_{y,2} = \frac{NL_0}{30}(4\theta_{y,2} - \theta_{y,1}) + \frac{2EI_y}{L_0}(2\theta_{y,2} + \theta_{y,1}) \tag{7}$$

The element's bowing effect is taken into account by the appearance of the axial force term N in the equations of moment (4)–(7) The local element's end scalar reactions so found are then transformed into global vector reactions forces by imposing static equilibrium to the element [15] or assuming equivalence of strain energy [16] thus obtaining the *global* shear force vector components (missing at a local reference frame level). With the global element's end reactions so found, an out-of-balance force $\overline{R}_i$ and out-of-balance moment $\overline{H}_i$ can be calculated for the generic ith node as vector summation of global reactions of the elements surrounding the node, plus external applied forces (and moments). Accordingly, the equilibrium geometry (nodes position and local frame orientations) such that the residuals $\overline{R}_i$ and $\overline{H}_i$ are null, can be found by implementation of a explicit resolution method such as Dynamic Relaxation (DR). The resolution of co-rotational beam-element formulation by DR method was first developed by Williams (as reported by Adriaenssens [14].

2.1.1. The dynamic relaxation

The DR method, firstly proposed by Day [17] and Otter [18], is a fictitious time-stepping scheme, where the positions of nodes representing a structural system are obtained by iterative numerical integration of Newton's second law of motion until the entire system reaches static equilibrium by the application of a viscous or kinetic [19] damping term. For a given structural system, the finding of the equilibrium geometry such that the residual forces and moments ($\overline{R}_i$ and $\overline{H}_i$) are null, can be pursued by implicit Finite Element analysis procedures (e.g. the well known Newton–Raphson method). However, an explicit Finite Element approach (such as DR in conjunction with the co-rotational formulation) allows the solution to converge independently of the magnitude of the initial deformed state, and thus is more suitable for form finding analyses involving large displacements. Moreover, since the DR operates at a vector level, it does not require the assembly and manipulation of a global stiffness matrix, hence it is relatively easy to implement and suitable for parallel computing [20]. In