Computers and Structures 154 (2015) 192–203

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

# On damping created by heterogeneous yielding in the numerical analysis of nonlinear reinforced concrete frame elements

Pierre Jehel [a,b,*], Régis Cottereau [a]

[a] MSSMat (UMR 8579) CNRS/CentraleSupélec, Grande voie des Vignes, 92290 Châtenay-Malabry, France
[b] Department of Civil Engineering and Engineering Mechanics, Columbia University, 630 SW Mudd, 500 West 120th Street, New York, NY 10027, USA

ARTICLE INFO

*Article history:*
Received 16 September 2014
Accepted 2 March 2015
Available online 21 April 2015

*Keywords:*
Damping
Concrete
Nonlinear constitutive relation
Material heterogeneity
Stochastic field

ABSTRACT

In the dynamic analysis of structural engineering systems, it is common practice to introduce damping models to reproduce experimentally observed features. These models, for instance Rayleigh damping, account for the damping sources in the system altogether and often lack physical basis. We report on an alternative path for reproducing damping coming from material nonlinear response through the consideration of the heterogeneous character of material mechanical properties. The parameterization of that heterogeneity is performed through a stochastic model. It is shown that such a variability creates the patterns in the concrete cyclic response that are classically regarded as source of damping.


## 1. Introduction

In the last few decades, a great deal of attention was paid to the comprehension and modeling of damping mechanisms in inelastic time-history analyses (ITHA) of concrete and reinforced concrete (RC) structures [1, Section 2.4]. Fig. 1, adapted from [2], shows the uniaxial cyclic compressive strain–stress ($E$–$\Sigma$) response measured on a concrete test specimen. Throughout this paper, the term "uniaxial" implies that there is only one loading direction and that the stress, respectively strain, of interest is the normal component of the stress, respectively strain, vector in the loading direction. In other words, when it comes to constitutive relation between stress and strain, the work presented thereafter is developed in a 1D setting. In Fig. 1, the so-called backbone curve, which is the envelope of the response (dashed line), shows the following phases: (i) an inelastic phase with positive slope ($E \leqslant 2.7 \times 10^{-3}$ for that particular example, where $E$ is the measured strain), and (ii) an inelastic phase with negative slope before the specimen collapses. For concrete, no elastic phase can really be identified, and hysteresis loops appear in unloading–reloading cycles even for limited strain amplitudes. Other salient features include: (i) a residual deformation after unloading, and (ii) a progressive degradation of the stiffness (slope of the unloading–loading segments). The hysteresis loops are one of the sources of the damping that is observed in free vibration recordings of concrete beams. Other sources include friction at joints [3] or at the concrete-steel interface in reinforced concrete [4]. These other sources of damping will not be discussed in this paper, where we will concentrate on material damping.

Most classical uniaxial constitutive models of concrete for numerical simulation do not dissipate any energy in unloading–reloading cycles (see e.g. Fig. 2 [top left]). It is then common practice to add a viscous damping model (Rayleigh damping) to the inelastic structural model, to reproduce phenomena that are experimentally observed at the structural level (decreasing amplitude of displacements in free vibration). However, Rayleigh damping is well known to lack physical justification, even when care is taken to avoid generating spurious damping forces [5,6].

Another class of approach aims at reproducing more precisely the features of Fig. 1 through elaborate inelastic constitutive relations. Fig. 2 shows typical examples of uniaxial relations found in the literature. The relation described in [7] [top left] defines different response phases for different strain intensities, with an additional coefficient to control the loss of stiffness. The constitutive relation described in [8] [bottom right] comes from a formulation developed in the framework of thermodynamics with internal variables. It reproduces damping features reasonably well, in particular for higher amplitudes, but requires the identification of a rather large number of parameters. These first two types of relations are somehow defined by parts for different loading regimes. They hence require a wider set of parameters and seem to contradict the seemingly smooth transition between regimes observed experimentally. The constitutive relation described in [9] [top

* Corresponding author at: MSSMat (UMR 8579) CNRS/CentraleSupélec, Grande voie des Vignes, 92290 Châtenay-Malabry, France.

*E-mail address:* pierre.jehel@centralesupelec.fr (P. Jehel).

http://dx.doi.org/10.1016/j.compstruc.2015.03.001
0045-7949/

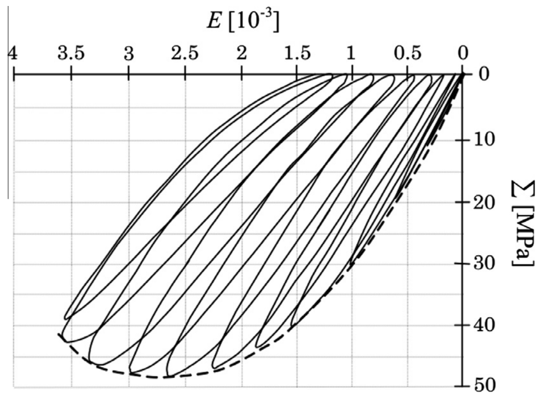


**Fig. 1.** Strain–stress concrete experimental response in pseudo-static cyclic uniaxial compressive loading (adapted from [2]). $\Sigma$ and $E$ are the homogeneous compression stress and strain in the concrete test specimen that are measured in the loading direction. $\Sigma$ and $E$ are spatial mean quantities in the sense that $\Sigma$ is computed as the load in the hydraulic cylinder of the testing machine divided by the area of the specimen cross section, and $E$ is computed as the displacement of the cylinder divided by the length of the concrete specimen.

right] is heuristically defined from a database of experiments. It reproduces unloading–reloading hysteresis mechanisms, but lacks a theoretical basis. Finally, the relation described in [10] [bottom left] is based on a physical model of damage and friction. It manages to dissipate energy in unloading–reloading cycles, but the lack of obvious physical meaning for some parameters can render their identification difficult.

The main purpose of this paper is to present a multi-scale stochastic nonlinear concrete model that can be accommodated in an efficient structural frame element (fiber element), and that participates to the overall structural damping in dynamic loading. In particular, this implies the developed concrete model be capable of representing hysteresis loops in unloading–reloading cycles at macro-scale (the scale where such behavior as in Fig. 1 can be observed). In this work, this is achieved by the introduction, at an underlying meso-scale, of spatial variability in the parameters. At meso-scale, an elasto-plastic response with linear kinematic hardening and heterogeneous yield stress is considered. This choice is mainly driven by its simplicity and its relevance is illustrated in the numerical applications.

The main issue with modeling the heterogeneity of the yield stress lies in the parameterization. On the one hand, local information on the heterogeneity of concrete is available at a scale that we wish to avoid (because of the associated computational costs). On the other hand, identification becomes extremely difficult when very fine models are considered. We therefore choose to model the heterogeneity of the yield stress through a stochastic model. Hence, only three parameters control that heterogeneity: a mean value, a variance, and a correlation length. The choice of parameterizing the fluctuating field of constitutive parameters by statistical quantities means that there might be fluctuations in the quantities of interest measured for different realizations of the random model. However, as will become apparent in the examples, some sort of homogenization comes in and these fluctuations can rightfully be ignored.

Several authors in the literature have considered random models of fluctuating nonlinear materials [11–14], in particular for concrete [15–21] or in the context of dynamic analysis [22–24]. We consider here a modeling framework that is a combination of ingredients found in several previous papers [25,26,15,21], with a fluctuating yield stress modeled as a random field with non-zero correlation length. However, the objective in these papers was to assess the influence of parameter uncertainty on some quantity of interest. An objective with the current paper is to observe the effect of randomness at a meso-scale on the nonlinear stress–strain relation at macro-scale. The work herein presented should therefore be seen as an innovative proposal for parameterization of a nonlinear stress–strain relation.

In Section 2, we recall the theoretical formulation of the inelastic beam model that will be used throughout this paper. The stochastic multi-scale constitutive relation developed to represent concrete cyclic behavior in reinforced concrete frame elements is introduced in Section 3. Concrete behavior at macro-scale is retrieved from the description of a meso-scale where elasto-plastic response with linear kinematic hardening and spatially variable yield stress is assumed. In particular, we emphasize in Sections 3.3 and 3.4 the heterogeneity of the yield stress and the parameterization of that heterogeneity through a random model. In Section 4, we report on the limiting case of vanishing correlation length and monotonic loading, for which several results can be derived analytically. Section 5 presents numerical applications of the model in the context of dynamic structural analysis of reinforced concrete frame elements.

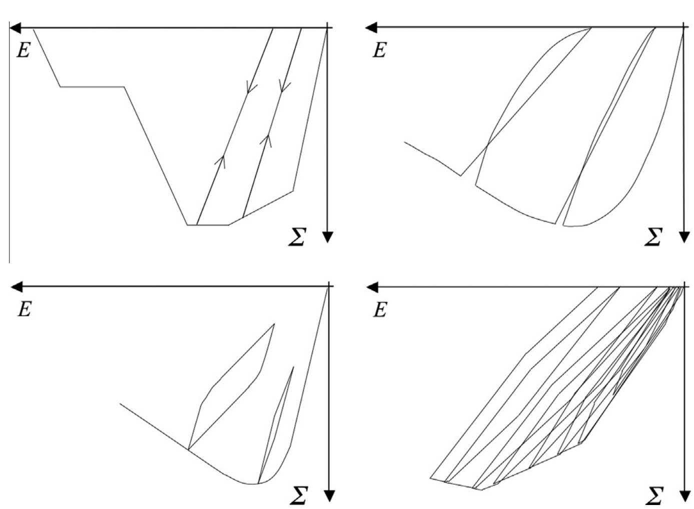


**Fig. 2.** Typical strain–stress relations in pseudo-static cyclic compressive loading for different models: [7] [top left], [9] [top right], [10] [bottom left], and [8] [bottom right].