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Cointegration, long-run structural modelling and weak exogeneity: Two models of the UK economy

Jan P.A.M. Jacobs [a], Kenneth F. Wallis [b,*]

[a] *University of Groningen, The Netherlands*

[b] *University of Warwick, United Kingdom*

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ABSTRACT

Cointegration ideas as introduced by Granger in 1981 are commonly embodied in empirical macroeconomic modelling through the vector error correction model (VECM). It has become common practice in these models to treat some variables as weakly exogenous, resulting in conditional VECMs. This paper studies the consequences of different approaches to weak exogeneity for the dynamic properties of such models, in the context of two models of the UK economy, one a national-economy model, the other the UK submodel of a global model. Impulse response and common trend analyses are shown to be sensitive to these assumptions and other specification choices.

1. Introduction

Cointegration ideas as introduced by Granger (1981) are commonly embodied in empirical macroeconomic modelling through the vector error correction model (VECM). The VECM representation of a dynamic system is obtained as a simple rearrangement of the vector autoregressive (VAR) model advocated by Sims (1980), once the variables in the VAR are cointegrated. Sims had argued that the structural identification of the then-existing simultaneous-equation macroeconometric models was incredible, and he proposed the alternative strategy of estimating the unrestricted reduced form, treating all variables as endogenous, namely the VAR. Having initially been banished from the scene, however, ideas of exogeneity and structural identification have gradually reappeared on stage in various guises. Thus it has become common practice in cointegrated VAR models to treat some variables as weakly exogenous, resulting in partial or conditional VECMs. And the recognition that, for policy analysis, VAR models still require identifying assumptions has resulted in a variety of ways of formulating "structural VAR" (SVAR) models. In a similar vein, the identification of multiple cointegrating relationships by restrictions drawn from economic theory, leaving the short-run dynamic and stochastic specification unrestricted, is called "long-run structural modelling" by Pesaran and Shin (2002). This approach is applied in the construction of a small quarterly model of the UK economy by Garratt et al. (2000; 2003; 2006, henceforth GLPS). Extended to a multi-country context, the same approach is applied in the construction of the global VAR (GVAR) model of Pesaran et al. (2004), further developed by Dees et al. (2007a,b).

The GLPS model features in our previous model comparison exercise (Jacobs and Wallis, 2005). It is used as an example of the SVAR style of modelling, for comparison with a modern example of the more traditional simultaneous-equation macroeconometric model (SEM). The two models under consideration differ appreciably in size, also in their treatment of exogeneity questions. The original VAR models were noticeably distinct from SEMs in abandoning the classification of variables as endogenous or exogenous, as noted above. In the closed economy context of much of the early empirical VAR analysis – the US economy, that is – this meant treating policy variables as endogenous, but here SEMs have followed suit, now containing policy reaction functions in place of their previous treatment of policy instruments as exogenous variables. In an open economy context, however, the distinction remains. The GLPS model treats variables describing the overseas economy as variables to be modelled in the same way as those describing the domestic economy, whereas in the UK SEM considered in our previous study the effect of the UK economy on the rest of the world is

* Corresponding address: Department of Economics, University of Warwick, Coventry CV4 7AL, United Kingdom. Tel.: +44 24 7652 3055, +44 24 7652 3026; fax: +44 24 7652 3032.
E-mail address: K.F.Wallis@warwick.ac.uk (K.F. Wallis).

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assumed to be negligible and "world" variables are treated as exogenous and are mostly unmodelled. The GVAR model, however, takes an intermediate position. Each national-economy or regional block of the GVAR model is a conditional VECM of similar dimension to the GLPS model of the UK economy, with some differences in the menu of variables. Unlike GLPS, however, the foreign variables in each separately estimated country submodel are treated as weakly exogenous. This difference between the GLPS model and the UK block of the GVAR model (henceforth GVAR(UK)) is noted in our previous article as a subject for future comparative research, which we undertake in the present paper. Different approaches to weak exogeneity questions have developed in the cointegration literature, and many associated econometric–theoretical issues have been addressed. However the impact of different weak exogeneity assumptions on the dynamic properties of the system appears not to have been studied hitherto. This paper presents such a study, in the context of two models of the UK economy which, while both representative of the VECM style of modelling, are rather different in their approach. We work with the published versions of the models, as estimated and tested by the respective modelling teams, varying only their treatment of exogeneity.

The remainder of the paper is organised as follows. Section 2 briefly reviews the formalities of the VAR–VECM modelling framework, the role of weak exogeneity assumptions and the conditional model, and different approaches to the specification of the associated marginal model. Section 3 uses the GLPS model of the UK economy to illustrate the effects of different weak exogeneity assumptions on the dynamic properties of a model, as revealed by its long-run multipliers and its impulse responses in a simple simulation exercise. Section 4 contrasts the treatment of weak exogeneity in the estimation and solution of the global VAR model, with special reference to its UK block, and reproduces a further simulation exercise. Section 5 compares the two models' common trends, where differences reflect specification choices other than the treatment of weak exogeneity. Section 6 concludes.

2. Cointegrated VARs and conditional VECMs

The VAR system is written

$$A(\mathrm{L})z_t = e_t, \tag{1}$$

where the matrix polynomial $A(\mathrm{L})$ has degree k and leading matrix equal to the identity matrix, reflecting the reduced-form nature of the system. Once the n variables in the vector z_t have been selected, with reference to the problem at hand, there is no prior classification as endogenous or exogenous; all are treated equally as variables of interest to be modelled. Impulse responses are calculated from the vector moving average representation

$$z_t = A(\mathrm{L})^{-1}e_t = C(\mathrm{L})e_t, \tag{2}$$

where the leading matrix in $C(\mathrm{L})$ is again the identity matrix. The elements of e_t are correlated, that is, $E(e_te_t') = \Omega$ is not diagonal, and Sims (1980) argued that it is useful to transform them to orthogonal form to be able to see the "distinct patterns of movement" of the system.

The VAR system (1) can be rearranged as

$$A^*(\mathrm{L})\Delta z_t = -\Pi z_{t-1} + e_t$$

where $\Pi = A(1)$ and the degree of $A^*(\mathrm{L})$ is $k-1$. If the elements of z_t are $I(1)$ and cointegrated with rank $(\Pi) = r,\ 0 < r < n$, then $\Pi = \alpha\beta'$ where α and β are $n \times r$ matrices of rank r, giving the VECM representation

$$A^*(\mathrm{L})\Delta z_t = -\alpha\beta' z_{t-1} + e_t. \tag{3}$$

Exact identification of β requires r restrictions on each of the r cointegrating vectors (columns of β), of which one is a normalization restriction and the other $r-1$ restrictions satisfy the identification rank condition. In the Wold representation of the differenced (stationary) variables

$$\Delta z_t = D(\mathrm{L})e_t \tag{4}$$

the matrix $D(1)$ of long-run multipliers corresponds to C_∞ in representation (2). It has rank $n-r$ and is given in Johansen's (1991) presentation of the Granger Representation Theorem as

$$D(1) = \beta_\perp[\alpha_\perp' A^*(1)\beta_\perp]^{-1}\alpha_\perp' \tag{5}$$

where the orthogonal complements $\alpha_\perp$ and $\beta_\perp$ are $n \times (n-r)$ matrices of rank $n-r$ such that $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$.

Various permanent-transitory decompositions follow from this representation. Stock and Watson (1988) show that, with r stationary linear combinations $\beta' z_t$, the $I(1)$ characteristics of z_t may be expressed in terms of $n-r$ "common trends" $\beta_\perp' z_t$. This formulation of the common trends as functions of the variables in the system has advantages for some purposes, although other formulations in terms of cumulated shocks are also available. The shocks that drive the common stochastic trends are the shocks $\alpha_\perp' e_t$, called permanent shocks, leaving r transitory shocks: since $\beta' D(1) = 0$, shocks to the cointegrating vectors have no permanent effects. Writing the Wold representation (4) as

$$\Delta z_t = D(\mathrm{L})H^{-1}He_t.$$

Levtchenkova et al. (1998) define a basic permanent-transitory decomposition as He_t, with the first $n-r$ elements permanent and the last r elements transitory, that is, $D(1)H^{-1}$ has its last r columns equal to zero. Then H has the form

$$H = \begin{bmatrix} \alpha_\perp' \\ \rho' \end{bmatrix}$$

for any $n \times r$ matrix ρ such that H is invertible, and Levtchenkova et al. discuss various possible choices of ρ. For example, Gonzalo and Granger (1995) take $\rho = \beta$.

Despite identification of the cointegrating vectors by restrictions on β, permanent-transitory decompositions require further structural identifying restrictions, or stories. Given β and an initial choice of $\beta_\perp$, note that $\beta'\beta_\perp P = 0$ for any nonsingular $(n-r)\times(n-r)$ matrix P. If $n-r > 1$, then identification of individual common trends $\beta_\perp' z_t$ requires restrictions on $\beta_\perp$ that make transformations $P'\beta_\perp' y_t$ inadmissible, whereas if $n-r = 1$, only a normalization restriction is required. Likewise, identifying individual permanent shocks requires further restrictions.

The conditional VECM model of a p-element subset y_t of the $n \times 1$ vector z_t is obtained if the remaining $q = n-p$ variables x_t can be treated as weakly exogenous. For this purpose it is convenient to rewrite the VECM representation (3) as

$$\Delta z_t = -\alpha\beta' z_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + e_t, \tag{6}$$

place the variables x_t in the last q positions of the vector z_t, and introduce conformable partitionings of relevant vectors and matrices as

$$z_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}, \qquad \alpha = \begin{bmatrix} \alpha_y \\ \alpha_x \end{bmatrix}, \qquad \Gamma_i = \begin{bmatrix} \Gamma_{yi} \\ \Gamma_{xi} \end{bmatrix},$$

$$e_t = \begin{bmatrix} e_{yt} \\ e_{xt} \end{bmatrix}, \qquad \Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}.$$

If $\alpha_x = 0$ then x_t is weakly exogenous and valid inference can proceed in the conditional model of y_t given x_t and the past, namely

$$\Delta y_t = \Lambda \Delta x_t - \alpha_y \beta' z_{t-1} + \sum_{i=1}^{k-1} \tilde{\Gamma}_{yi} \Delta z_{t-i} + \tilde{e}_{yt} \tag{7}$$

where $\Lambda = \Omega_{yx}\Omega_{xx}^{-1}$, $\tilde{\Gamma}_{yi} = \Gamma_{yi} - \Lambda\Gamma_{xi}$ and $\tilde{e}_{yt} = e_{yt} - \Lambda e_{xt}$ (Johansen, 1995, ch. 8).