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Semiparametric inference in multivariate fractionally cointegrated systems[☆]

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ABSTRACT

A semiparametric multivariate fractionally cointegrated system is considered, integration orders possibly being unknown and $I(0)$ unobservable inputs having nonparametric spectral density. Two estimates of the vector of cointegrating parameters ν are considered. One involves inverse spectral weighting and the other is unweighted but uses a spectral estimate at frequency zero. Both corresponding Wald statistics for testing linear restrictions on ν are shown to have a standard null χ^2 limit distribution under quite general conditions. Notably, this outcome is irrespective of whether cointegrating relations are "strong" (when the difference between integration orders of observables and cointegrating errors exceeds 1/2), or "weak" (when that difference is less than 1/2), or when both cases are involved. Finite-sample properties are examined in a Monte Carlo study and an empirical example is presented.

1. Introduction

Semiparametric modelling has become popular in cointegration analysis of $I(1)$ time series with $I(0)$ cointegrating errors. In the simplest parametric setting, observables follow a random walk and cointegrating errors are serially uncorrelated. Vector autoregressive (VAR) extensions have been developed (e.g. Johansen, 1991), but optimal inference on the unknown cointegrating relations loses validity if the VAR order is under-specified, or if the process lies outside the VAR class. Phillips and Hansen (1990) and Phillips (1991a) and others showed that one can do as well allowing the $I(0)$ inputs to have nonparametric autocorrelation, under suitable conditions on the bandwidth employed in the smoothed nonparametric spectrum estimate.

Another source of possible misspecification is the basic $I(1)/I(0)$ framework itself. Recently, optimal inference has developed in a fractional setting (see e.g. Jeganathan, 1999; Robinson and Hualde, 2003). Here, integration orders were allowed to be unknown, to non-trivially generalize the $I(1)/I(0)$ assumption, but theory was developed only in a fully parametric setting, incurring the familiar concern about misspecification, and just for a bivariate situation, hence avoiding the complexity in simultaneously dealing with multiple cointegrating relations where integration orders of errors may differ. Dolado and Marmol (1996) and Kim and Phillips (2000) allowed nonparametric autocorrelation in $I(0)$ inputs but, respectively assuming knowledge of integration orders, and proposing sub-optimal procedures.

The present paper develops inference on cointegrating relations in a semiparametric fractional setting, with unknown integration orders. To describe our model, we introduce the following definitions corresponding to ones in Robinson and Hualde (2003) (hereafter RH). For any scalar or vector sequence $v_t, t = 0, \pm 1, \ldots$, we denote

$$v_t^{\#} = v_t 1(t > 0),$$

where $1(\cdot)$ is the indicator function. Defining the difference operator $\Delta = 1 - L$, where L is the lag operator, the fractional difference operator is given formally, for any real α, $\alpha \neq -1, -2, \ldots$, by

$$\Delta^{-\alpha} = \sum_{j=0}^{\infty} a_j(\alpha) L^j, \quad a_j(\alpha) = \frac{\Gamma(j+\alpha)}{\Gamma(\alpha)\Gamma(j+1)},$$

with Γ denoting the gamma function. Denoting by a_{it} the ith component of an arbitrary vector process a_t, we say that a scalar process ζ_t is integrated of order d, $\zeta_t \sim I(d)$, if for any $l \times 1$ $(l < \infty)$ covariance stationary process ξ_t whose spectral density matrix is continuous and nonsingular at all frequencies,

$$\zeta_t = \sum_{k=1}^{l} \Delta^{-d_k} \xi_{kt}^{\#}, \tag{1}$$

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for $d = \max_{1\le k\le l} d_k$. Robinson and Gerolimetto (2006) refer to each summand of (1) as "basic fractional", so our $I(d)$ process, like theirs, is a linear combination of "basic fractional" processes, with maximal order d. We say that a vector process is $I(d)$ if at least one of its components is $I(d)$, the rest having integration orders no greater than d. This definition is identical to that of Hualde (2008) and resembles that of Johansen (2008), which allows $I(d)$ vectors to have individual components of smaller order than d.

For any $r_0 \times 1$ vector $d = (d_1, \ldots, d_{r_0})'$, the prime denoting transposition, denote $\Delta(d) = \operatorname{diag}\left\{\Delta^{d_1}, \ldots, \Delta^{d_{r_0}}\right\}$. Let u_t, $t = 0, \pm 1, \ldots$, be an $r_0 \times 1$ covariance stationary unobservable process with zero mean and nonparametric spectral density matrix $f(\lambda)$, given by

$$E(u_0 u_j') = \int_{-\pi}^{\pi} e^{ij\lambda} f(\lambda) d\lambda,$$

that is at least continuous and nonsingular at all frequencies. For a $r_0 \times r_0$ nonsingular matrix Υ and a $r_0 \times 1$ possibly unknown vector $\delta = (\delta_1, \ldots, \delta_{r_0})'$, we define the $r_0 \times 1$ vector observable process z_t, $t = 0, \pm 1$, by

$$\Upsilon z_t = \Delta^{-1}(\delta) u_t^{\#}, \tag{2}$$

where, without loss of generality we set

$$0 \le \delta_1 \le \delta_2 \le \cdots \le \delta_{r_0}, \tag{3}$$

which implies $z_t \sim I(\delta_{r_0})$. We exclude antipersistent processes (having negative integration order), which seem to have limited economic relevance, except in an over-differencing context.

The system (2) is the general framework in which we will discuss the cointegrating properties of z_t. We say that a vector $\xi_t \sim I(d)$ is cointegrated if there exists a vector $\alpha \neq 0$ such that $\alpha' \xi_t \sim I(c)$ with $c < d$. This definition covers the standard notion of cointegration where the observables share the same integration order (as in, e.g., Engle and Granger, 1987), but also many others, where observables with different integration orders might combine in the cointegrating relation. The definition is identical to that in Hualde (2008), which is similar to that of Johansen (1996), and more general than those in Flores and Szafarz (1996) and Robinson and Yajima (2002) or Robinson and Marinucci (2003).

The system (2) is very general, but unless δ and Υ are further restricted, it does not necessarily represent a cointegrated model and moreover, is nonidentifiable. As will be seen in Section 2, additional restrictions on δ and Υ are motivated by the particular cointegrating structure which characterizes our observables. Apart from other possible constraints, we will assume in all cases that Υ is upper-triangular with 1's in the main diagonal. We justify below that there is no loss of generality in this restriction, especially in connection to (3). As a very relevant improvement over existing literature we do not assume that all observables share the same integration order (unlike, e.g., Kim and Phillips, 2000; Chen and Hurvich, 2003, 2006; and effectively also Robinson and Yajima, 2002), although this possibility is considered and materializes if the last column of Υ^{-1} contains no zeroes. In addition, we allow the orders of the cointegrating errors to possibly vary, unlike in the standard cointegration literature where all these orders are assumed zero. We will deal with the case of an arbitrary cointegrating rank (that is the number of linearly independent cointegrating vectors) $r_1 \in \{1, \ldots, r_0 - 1\}$, but our results are new even if $r_1 = 1$, for any $r_0 \ge 2$. Given there are no prior restrictions on f, those which will be imposed on Υ and δ ensure identification, and imply that (given consistent estimates of Υ and δ) consistent estimation of f is possible, which is fundamental to our approach.

The truncation in (2) is motivated by systems where at least δ_{r_0} falls in the nonstationary region, $\delta_{r_0} > 1/2$. This version of fractional integration ("Type II" process) and cointegration accords with that in RH. An alternative one ("Type I" process), for which the procedures developed below nevertheless apply, was used by Dolado and Marmol (1996), Jeganathan (1999) and Kim and Phillips (2000). None of these references covers δ_{r_0} within the stationary region, $\delta_{r_0} \in (0, 1/2)$, which will be permitted by our setting; in this case we say that our relations display "stationary cointegration". This arose in Robinson (1994a), and it has been stressed in a finance context by Bandi and Perron (2004) and Christensen and Nielsen (2006). A larger class (where $\delta_{r_0} > 1/2$ is possible) consists of cases where the cointegrating gap (the difference between the integration order of the observables and cointegrating error) falls in the $(0, 1/2)$ region, which we denote "weak cointegration". Empirical evidence of this, with nonstationary observables, was found by Robinson and Marinucci (2003), and it has been further discussed by Hualde and Robinson (2007). The case where the gap is greater than 1/2, which includes the usual $I(1)/I(0)$ situation, is called "strong cointegration".

It is desired to conduct inference on the unknown elements of Υ, in the presence of unknown δ. The present paper does not merely extend nontrivially the bivariate model in RH to a richer multivariate framework, and allow also for nonparametric f, but simultaneously covers relations of weak and strong cointegration, which, as we understand, has not been attempted before. While asymptotics for point estimates of unknown parameters in Υ differ significantly across these cases, the same rules of inference prevail throughout, with the same Wald test statistic (for a linear hypothesis on these parameters) having a null limit χ^2 distribution. The borderline situation between strong and weak cointegration, with a cointegrating gap of 1/2, will be excluded largely because it seems too special to warrant the space necessary to present the somewhat separate technical treatment that it would require. However, while the convergence rate of our estimates differs from those under both strong and weak cointegration, it seems that the same limit distribution for the Wald statistic will still hold, so that slight limitation of our analysis can be dispensed with.

We find it convenient to treat the nonparametric autocorrelation in the frequency domain. This prompts consideration of two alternative methods of estimating and testing hypotheses on ν. One involves a ratio of weighted periodogram averages either across all frequencies in the Nyquist band, or only over those within a shrinking neighbourhood of zero frequency. The weighting is inverse with respect to smoothed estimates of f. Because of the concentration of spectral mass around zero frequency, where f changes little, computationally simpler statistics, with the same asymptotic properties, replace the weights by multiplicative factors based on an estimate of $f(0)$.

The plan of the paper is as follows. In Section 2 additional restrictions on δ and Υ and estimates of the cointegrating matrix and test statistics will be introduced. Regularity conditions and asymptotic properties are presented in Section 3. Section 4 contains a Monte Carlo study of finite-sample behaviour, and Section 5, the analysis of an empirical example. Some concluding remarks are made in Section 6. Proofs are relegated to an Appendix.

2. Estimation of cointegrating parameters and test statistics

As previously mentioned, we need to introduce additional restrictions on Υ, δ, which ensure cointegration and identification, and then propose estimates of the unrestricted parameters in Υ. Our basic assumption (which materializes in Assumption 1) is that the cointegrating properties of z_t are characterized by the following structure. First, $S_{r_1}^{(1)} \subset \mathbb{R}^{r_0}$ represents the cointegrating space of dimension $r_1 < r_0$. This implies the existence of a full rank $r_0 \times r_1$ matrix $\beta(1)$ (whose columns are cointegrating vectors) such that $\beta(1)' z_t \sim I(\delta_{r_1})$, with $\delta_{r_1} < \delta_{r_0}$. Hualde (2008) shows that δ_{r_1} is uniquely identified, in the sense that for any other arbitrary $r_0 \times$