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Dynamics of beams on non-uniform nonlinear foundations subjected to moving loads

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ABSTRACT

This paper presents a study on the dynamic response of beams on nonlinear elastic foundations, subjected to moving loads. The effects of the load's intensity and velocity and of the foundation's stiffness are investigated. Critical velocities are determined for beams on uniform foundations or foundations composed of two sub-domains with different values of stiffness, with or without viscous damping. In the case of non-uniform linear foundations, a very good agreement is obtained between the results of the model and available analytical solutions. The goal of this study is to generalize, for more realistic foundation behaviors, the analyses obtained by other authors.

1. Introduction

The study of the dynamic response of a beam on an elastic foundation subjected to a moving load is of special interest in the design and management of high-speed rail tracks. The first commercial high-speed trains appeared in Japan in 1964 and since then have spread through the industrialized countries, allowing intercity connections at average speeds of 300 km/h. For travelling distances between 250 and 900 km, the high speed train can actually be faster than the airplane, for which the time spent going to and coming from the airport, in security control and in boarding can significantly slow down the passengers' airplane trip. The relevance of the high-speed train can grow even more in the future since the peak velocities of such trains have steadily increased over the past decades. The present record belongs to the French TGV, which reached in 2007 the speed of 574.8 km/h in a test run [1].

It is well know that constant loads moving with a constant velocity on uniform beams supported by uniform foundations may lead to significantly different dynamic behaviors depending on the velocity magnitude; for some velocity ranges the oscillation amplitudes may become very large, thus endangering the structural and passengers' safety. Hence, it is interesting to study the response of beams on elastic foundations under moving loads in order to mitigate the above mentioned effects.

The response of beams subjected to moving loads has been investigated over the years by several authors. Firstly, it is worth mentioning the pioneer works of Krylov [2] and Timoshenko [3] in the beginning of the last century, in which dynamic stresses are determined for simply supported beams under a moving load. Later on, Inglis [4], Lowan [5] and Frýba [6] studied the transverse vibrations using analytical methods in which the deflections are the result of the sum of multiple normal modes. The static problem of a beam on a linear elastic foundation was approached by Hetenyi [7]. Timoshenko et al. [8] analytically solved the free vibration problem of a beam on an elastic foundation.

Various foundation models such as Winkler, Pasternak, Vlasov or Reissner and either Euler–Bernoulli or Timoshenko beams have been used [9–13]. Nonlinear models where the foundation reaction has a cubic dependence on the displacement [14,15] or where the foundation is considered as tensionless have also been considered [16–18]. The response of beams resting on elastic foundations and subjected to moving oscillators has also been investigated [19–21].

The finite element method was also used to study the behavior of beams on elastic foundations subjected to moving loads. Thambiratnam and Zhuge [22] analyzed the influence of the beam's length, the travelling speed of the load and of the foundation's stiffness on the dynamic amplifications of mid-span deflections and stresses. Andersen et al. [23] and Nguyen and Duhamel [24] proposed alternative formulations of the finite element method in which the dynamic equations are reformulated in a reference frame which follows the load. Senalp et al. [25] used the Galerkin method and the finite element method in the analysis of mid-span deflections of a finite beam on a nonlinear foundation. The finite element method was also used to determine the

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response of beams resting on elastic foundations and subjected to moving oscillators [26].

Recently, Dimitrovová and Rodrigues [27] determined analytically the critical velocities which cause resonance of finite and infinite beams under moving loads, on uniform or non-uniform linear elastic foundations, with or without viscous damping. However, under dynamic loading, the support structure of a railway track may be highly nonlinear because of the hardening characteristic of the ballast, as shown by Dahlberg [28]. This paper presents the results obtained from the development of a finite element program in MatLab environment [29] used for the dynamic analysis of beams on a nonlinear elastic foundation subjected to a moving load. The use of the finite element method has the advantage of solving nonlinear problems for which there are no analytical solutions. Using this program, critical velocities are determined for beams on nonlinear uniform foundations or nonlinear foundations composed of two sub-domains with different values of stiffness, with or without damping. The effects of the load's intensity and velocity and of the foundation's stiffness are investigated. In the case of non-uniform linear foundations, a very good agreement is obtained between the results of the model and the analytical results available in the literature [27]. The goal of this study is to generalize, for more realistic (nonlinear) foundation behaviors, the analyses in [27]. This kind of studies are preliminary in addressing the behavior of high-speed railway infrastructures.

2. Finite element method formulation

We consider an Euler–Bernoulli beam finite element, of uniform height h and length l, on an elastic foundation defined between two generic nodes i and j, as shown in Fig. 1. The vector of the (four) generalized displacements (degrees of freedom) of the finite element is defined as

$$\mathbf{q}^e = \{q_1\ q_2\ q_3\ q_4\}^{\mathrm{T}}. \tag{1}$$

The transversal displacement of the beam is defined by

$$w(x) = [N_1(x) \quad N_2(x) \quad N_3(x) \quad N_4(x)] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \mathbf{N}^e(x)\mathbf{q}^e, \tag{2}$$

where $\mathbf{N}^e(x) = \mathbf{A}(x)\mathbf{D}^{-1}$ with $\mathbf{A}(x) = [1\ x\ x^2\ x^3]$ and

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l & l^2 & l^3 \\ 0 & 1 & 2l & 3l^2 \end{bmatrix}. \tag{3}$$

The shape functions are defined by $N_1(x) = 1 - 3\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)^3$, $N_2(x) = x\left(1 - \frac{x}{l}\right)^2$, $N_3(x) = 3\left(\frac{x}{l}\right)^2 - 2\left(\frac{x}{l}\right)^3$ and $N_4(x) = x\left[\left(\frac{x}{l}\right)^2 - \frac{x}{l}\right]$.

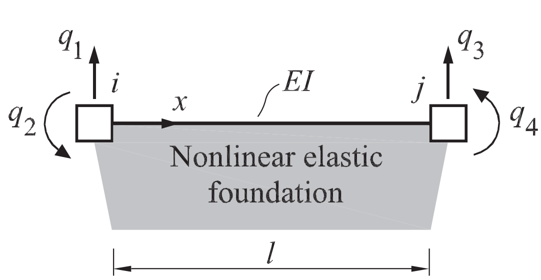

Fig. 1. Euler–Bernoulli beam finite element on an elastic foundation.

Neglecting the axial and shear strains and denoting by EI the beam's bending stiffness, the elastic strain energy of the finite element is given by

$$U_{\mathrm{b}} = \frac{1}{2}\int_0^l w''(x)M(x)\ \mathrm{d}x. \tag{4}$$

where

$$w''(x) = \mathbf{B}(x)\mathbf{D}^{-1}\mathbf{q}^e, \tag{5}$$

and

$$M(x) = EI\ \mathbf{B}(x)\mathbf{D}^{-1}\mathbf{q}^e \tag{6}$$

is the bending moment in the finite element. In (5) and (6), $\mathbf{B}(x) = \mathbf{A}''(x)$. Substituting (5) and (6) in (4) we obtain

$$U_{\mathrm{b}} = \frac{1}{2}(\mathbf{q}^e)^{\mathrm{T}}\mathbf{K}_{\mathrm{b}}^e\mathbf{q}^e. \tag{7}$$

where

$$\mathbf{K}_{\mathrm{b}}^e = \frac{\partial^2 U_{\mathrm{b}}}{\partial\mathbf{q}^e\partial\mathbf{q}^e} = \mathbf{D}^{-\mathrm{T}}\int_0^l \mathbf{B}^{\mathrm{T}}(x)\ EI\ \mathbf{B}(x)\ \mathrm{d}x\ \mathbf{D}^{-1} \tag{8}$$

is the elementary beam's stiffness matrix. Although the use of the Euler–Bernoulli beam model instead of the Timoshenko model may be seen as a limitation of this study, its use is justified by the fact that the differences between the results obtained with these models are negligible for slender beams (see for example [27]).

Dahlberg [28] showed that, under dynamic loading, the support structure of a railway track is highly nonlinear because of the hardening characteristic of the ballast. Fig. 2 shows the force–displacement relation of the soil under the beam used in this formulation. The foundation reaction force per unit length F_{f} is given by

$$F_{\mathrm{f}} = F_{\mathrm{l}} + F_{\mathrm{nl}} = k_{\mathrm{l}}w + k_{\mathrm{nl}}w^3. \tag{9}$$

This nonlinear relation represents a more realistic approximation of the behavior of the foundation of a railway track than the one used in [27] where just the first (linear) term of the right hand side of (9) is present. The elastic strain energy, per unit length, of the foundation is given by

$$u_{\mathrm{f}} = \int_0^w F_{\mathrm{f}}\ \mathrm{d}w = \frac{1}{2}\ k_{\mathrm{l}}\ w^2 + \frac{1}{4}\ k_{\mathrm{nl}}\ w^4, \tag{10}$$

thus, the elastic strain energy of the foundation underneath the finite element is given by

$$U_{\mathrm{f}} = \int_0^l u_{\mathrm{f}}\ \mathrm{d}x = \int_0^l \left(\frac{1}{2}\ k_{\mathrm{l}}\ w^2 + \frac{1}{4}\ k_{\mathrm{nl}}\ w^4\right)\ \mathrm{d}x. \tag{11}$$

Substituting (2) in (11) we get

$$U_{\mathrm{f}} = \frac{1}{2}(\mathbf{q}^e)^{\mathrm{T}}\mathbf{K}_{\mathrm{l}}^e\mathbf{q}^e + \frac{1}{4}\int_0^l k_{\mathrm{nl}}\left[\mathbf{A}(x)\mathbf{D}^{-1}\mathbf{q}^e\right]^4\ \mathrm{d}x, \tag{12}$$

where

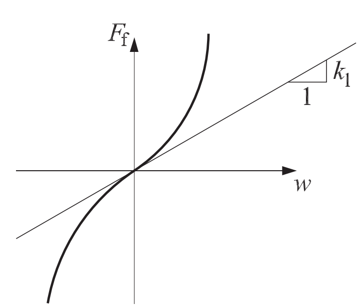

Fig. 2. Graph of the force–displacement relation of a nonlinear elastic foundation.