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Nonparametric statistical formulations for structural health monitoring

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ABSTRACT

This paper presents a group of nonparametric statistical formulations for structural health monitoring (SHM). Vibration response data are first represented by the coefficients of a series of fitted autoregressive (AR) models in the time domain or by the averages of binned power spectral density (PSD) estimates in the frequency domain. Three types of statistical hypotheses are then formulated and tested by nonparametric techniques to monitor these characteristics. Specifically, two-sample Kolmogorov–Smirnov test, Mann–Whitney test, and Mood test are used in this study. For each type of hypothesis formulation, a function of the resulting *P*-values is used to define a damage indicator profile (DIP) whereby damage locations are identified. The highlight of these formulations is that, due to their nonparametric nature, they do not require a particular functional form for the probability distribution of the underlying population of an extracted vibration response data characteristic. Two numerically simulated case studies, i.e., a 20-degree-of-freedom system and a hyperbolic paraboloid roof shell, demonstrate the efficacy of the proposed nonparametric SHM formulations. Multiple damage locations are also considered in the case studies.

1. Introduction

The challenges of improving structural safety have been stimulating continuous interest among civil engineering researchers to further the state of the art in the related areas of design, construction, inspection and retrofitting techniques. Particularly, the last several years have seen rapid research development in vibration-based structural health monitoring (SHM) [1,2]. This is partially due to its more effective performance compared with conventional methods such as visual inspection and its automatic nature in implementation, which make it suitable for various complicated applications along with sophisticated computing hardware. Over the years a large variety of SHM schemes have been proposed. Salawu [3] reviewed the damage detection methods using modal frequencies. Methods based on mode shapes [4,5], frequency response functions (FRFs) [6], flexibility [7,8], optimization algorithms [9], as well as advanced signal processing techniques [10–12] were also developed. Correspondingly, a range of structure types have been investigated, including frames [13–15], bridges [16–19], wind turbines [20,21], a masonry wall [22], and a full-scale retrofitted building [23]. Detailed reviews of the relevant literature were documented by Doebling et al. [24], Carden and Fanning [25], and Chan and Thambiratnam [2].

Recently, there have been increasing research efforts to address the SHM issues from a probabilistic and statistical point of view. Vanik et al. [26] formulated a Bayesian probabilistic SHM approach based on the probabilistic model updating scheme developed by Beck and Katafygiotis [27] and Katafygiotis and Beck [28]. Nair et al. [29] proposed a statistical damage detection and locating algorithm where time series models are fitted to develop a damage-sensitive data characteristic and the related damage locating indices. The differences in the means of the damage-sensitive data characteristics are checked by invoking *t*-test. Giraldo et al. [30] constructed a statistical SHM scheme whereby varying environmental conditions can be allowed for. Nair and Kiremidjian [31] addressed the SHM issues by Gaussian mixture models (GMMs). A multivariate statistical approach has more recently been designed by Wang and Ong [32] in which both time- and frequency-domain data can be utilized. Other relevant work includes Worden et al. [33], Kullaa [34], Lam et al. [35], Casciati [36], Posenato et al. [37], Lanata and Schoefs [38], and Döhler et al. [39].

The continued research efforts among the SHM community have been yielding a variety of damage-sensitive vibration response data characteristics with either deterministic or stochastic nature. This wide family of vibration response data characteristics provides the flexibility in selecting appropriate ones for various application

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Nomenclature

$\mathbf{A}_\mathrm{c}$, $\mathbf{A}_\mathrm{r}$ acceleration time histories in the current state, and those in the reference state
B backward shift operator
${}_\mathrm{c}\mathbf{a}_i$, ${}_\mathrm{r}\mathbf{a}_i$ acceleration time history at the *i*th output DOF in the current state, and that in the reference state
$\mathbf{D}_\mathrm{I}$, $\mathbf{D}_\mathrm{II}$, $\mathbf{D}_\mathrm{III}$ DIPs based on Formulations I, II, and III, respectively
$F_{{}_\mathrm{c}Z_{jl}}(\cdot), F_{{}_\mathrm{r}Z_{jl}}(\cdot)$ CDFs of ${}_\mathrm{c}Z_{jl}$ and ${}_\mathrm{r}Z_{jl}$, respectively
$\mathbf{f}$ equally-spaced frequencies corresponding to estimated PSDs
f_s sampling frequency for the acceleration time histories
$g_\mathrm{I}(\cdot)$, $g_\mathrm{II}(\cdot)$, $g_\mathrm{III}(\cdot)$ predefined functions for DIPs based on Formulations I, II, and III, respectively
H_0, H_1 null hypothesis and alternative hypothesis, respectively
I_c, I_r number of data points in each acceleration time history in the current state, and that in the reference state
M_c, M_r number of time windows for each acceleration time history in the current state, and that in the reference state
M'_c, M'_r number of time windows for each acceleration time history in the current state, and that in the reference state for AR model based formulation
M''_c, M''_r number of time windows for each acceleration time history in the current state, and that in the reference state for PSD estimate based formulation
m number of entries in $\mathbf{f}$, ${}_\mathrm{r}\mathbf{S}_{kj}$, or ${}_\mathrm{c}\mathbf{S}_{kj}$
m_1, m_2 sequence number of the entry in ${}_\mathrm{r}\mathbf{S}_{kj}$ (or ${}_\mathrm{c}\mathbf{S}_{kj}$) taken as the first entry in ${}_\mathrm{r}\mathbf{S}'_{kj}$ (or ${}_\mathrm{c}\mathbf{S}'_{kj}$), and that for the last entry in ${}_\mathrm{r}\mathbf{S}'_{kj}$ (or ${}_\mathrm{c}\mathbf{S}'_{kj}$)
N number of PSD-estimate data points in each frequency bin
n number of output DOFs
${}_\mathrm{I}P_{jl}$, ${}_\mathrm{II}P_{jl}$, ${}_\mathrm{III}P_{jl}$ *P*-values of the hypothesis tests based on ${}_\mathrm{r}z_{kjl}$ and ${}_\mathrm{c}z_{kjl}$, respectively corresponding to Formulations I, II, and III
p generic symbol for p' and p''
p' order of the AR models fitted to the data points in the time windows
p'' number of frequency bins used to partition a PSD estimate
$\mathbf{S}_\mathrm{c}$, $\mathbf{S}_\mathrm{r}$ estimated PSDs in the current state, and those in the reference state
${}_\mathrm{c}\mathbf{S}_{kj}$, ${}_\mathrm{r}\mathbf{S}_{kj}$ estimated PSDs based on the data points in the *k*th time window of the acceleration time history at the *j*th output DOF in the current state, and those in the reference state
${}_\mathrm{c}\mathbf{S}'_{kj}$, ${}_\mathrm{r}\mathbf{S}'_{kj}$ estimated PSDs corresponding to the frequency range of interest based on the data points in the *k*th time window of the acceleration time history at the *j*th output DOF in the current state, and those in the reference state
${}_\mathrm{c}\bar{\mathbf{S}}_{kj}$, ${}_\mathrm{r}\bar{\mathbf{S}}_{kj}$ average of the PSD-estimate data points based on the data points in the *k*th time window of the acceleration time history at the *j*th output DOF in the current state, and that in the reference state
${}_\mathrm{c}\bar{S}_{kjl}$, ${}_\mathrm{r}\bar{S}_{kjl}$ average of the PSD-estimate data points in the *l*th frequency bin based on the data points in the *k*th time window of the acceleration time history at the *j*th output DOF in the current state, and that in the reference state
s' number of data points in each time window for AR model based formulation
s'' number of data points in each time window for PSD estimate based formulation
${}_\mathrm{c}Z_{jl}$, ${}_\mathrm{r}Z_{jl}$ populations from which ${}_\mathrm{c}z_{kjl}$ and ${}_\mathrm{r}z_{kjl}$ are taken, respectively
${}_\mathrm{c}z_{kjl}$, ${}_\mathrm{r}z_{kjl}$ *l*th data point of the vibration response data characteristic based on the data points in the *k*th time window of the acceleration time history at the *j*th output DOF in the current state, and that in the reference state
${}_\mathrm{c}\varepsilon_{ij}$, ${}_\mathrm{r}\varepsilon_{ij}$ realizations of white noise processes with zero means in the current state, and those in the reference state
$\boldsymbol{\eta}_\mathrm{I}$, $\boldsymbol{\eta}_\mathrm{II}$, $\boldsymbol{\eta}_\mathrm{III}$ parameter vectors in $g_\mathrm{I}(\cdot)$, $g_\mathrm{II}(\cdot)$, and $g_\mathrm{III}(\cdot)$, respectively
θ_{jl} shift parameter in Formulation II hypotheses, or scale factor in Formulation III hypotheses
μ mean of the underlying random process corresponding to the acceleration data points in a time window
${}_\mathrm{c}\Phi_{kj}(\cdot)$, ${}_\mathrm{r}\Phi_{kj}(\cdot)$ function of B used to define AR models in the current state, and that in the reference state
${}_\mathrm{c}\phi_{kjl}$, ${}_\mathrm{r}\phi_{kjl}$ *l*th coefficient of the AR model fitted to the data points in the *k*th time window of the acceleration time history at the *j*th output DOF in the current state, and that in the reference state

scenarios, i.e., different structures, damage types, response data available, etc. It, on the other hand, brings the challenges for the applicability of subsequent statistical analyses. For instance, it is not uncommon that the samples involved in a statistical SHM scheme are assumed to be taken from a population having a normal distribution. Thus this SHM scheme might not be applicable to the vibration response data characteristics constructed based on a population severely deviating from normality. More generally, for a statistical SHM scheme sensitive to a parametric-statistical-model requirement to work properly, the involved random variables need to actually have the assumed probability distribution, or at least must not be significantly different from it. It follows that a nonparametric statistical SHM scheme would be preferable in the sense that, besides having the capacity of explicitly addressing the involved uncertainty as a parametric scheme does, a nonparametric one does not need to specify functional forms for the probability distributions of the populations based on which the vibration response data characteristics are obtained. Accordingly, an attempt is made in this study to formulate a nonparametric statistical SHM framework. In the following sections, the formulations of the proposed framework are presented, along with two case studies to demonstrate its efficacy.

2. Formulations of the nonparametric statistical SHM framework

2.1. Extracting the damage-sensitive vibration response data characteristics: time- and frequency-domain representations

For the structure being monitored, the state in which the structure is undamaged or the state which is deemed to be a benchmark for the subsequent states to be compared with may be defined as the reference state. The state wherein the structural condition needs to be investigated is labeled as the current state. In the dynamic testing, excitation forces are generated by Gaussian white noise processes, and the structure is instrumented with accelerom-