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# On the statistical identification of DSGE models[☆]

Agostino Consolo[a], Carlo A. Favero[a,b,c,d,*], Alessia Paccagnini[a]

[a] IGIER, Università Bocconi, Italy
[b] AMeN, Spain
[c] Department of Finance, Universita' Bocconi, Italy
[d] CEPR, UK

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ABSTRACT

Dynamic Stochastic General Equilibrium (DSGE) models are now considered attractive by the profession not only from the theoretical perspective but also from an empirical standpoint. As a consequence of this development, methods for diagnosing the fit of these models are being proposed and implemented. In this article we illustrate how the concept of statistical identification, that was introduced and used by Spanos [Spanos, Aris, 1990. The simultaneous-equations model revisited: Statistical adequacy and identification. Journal of Econometrics 44, 87–105] to criticize traditional evaluation methods of Cowles Commission models, could be relevant for DSGE models. We conclude that the recently proposed model evaluation method, based on the DSGE–VAR($\lambda$), might not satisfy the condition for statistical identification. However, our application also shows that the adoption of a FAVAR as a statistically identified benchmark leaves unaltered the support of the data for the DSGE model and that a DSGE–FAVAR can be an optimal forecasting model.



## 1. Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are now considered attractive by the profession not only from the theoretical perspective but also for empirical analysis and for econometric policy simulation.[1] Model evaluation is an issue of crucial importance before policy simulation. Therefore, methods for diagnosing the fit of these models are being proposed and implemented. This article illustrates how the concept of statistical identification, originally introduced to criticize traditional evaluation methods of Cowles Commission models, could also be applied to the diagnostic tools recently proposed for DSGE models.

The concept of statistical identification has been introduced by Spanos (1990). Structural models can be viewed statistically as a reparameterization, possibly (in case of over-identified models) with restrictions, of the reduced form. Spanos distinguishes between structural identification and statistical identification. Structural identification refers to the uniqueness of the structural parameters, as defined by the reparameterization and restriction mapping from the statistical parameters in a reduced form, while statistical identification refers to the choice of a well-defined statistical model as a reduced form. Diagnostics for model evaluation are constructed in the Cowles commission tradition in a way that is closely related to the solution of the identification problem. In fact, in the (very common) case of over-identified models, a test of the validity of the over-identifying restrictions can be constructed by comparing the restricted reduced form implied by the structural model with the reduced form implied by the just-identified model in which each endogenous variable depends on all exogenous variables with unrestricted coefficients. The statistics are derived in Anderson and Rubin (1949) and Basmann (1960). The logic of the test attributes a central role to

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* Corresponding author at: IGIER, Università Bocconi, Italy. Tel.: +39 0258363306; fax: +39 0258363302.
*E-mail address:* carlo.favero@unibocconi.it (C.A. Favero).

[1] See An and Schorfheide (2006) and the JBES Invited address presented at the Joint Statistical Meeting 2006 "On the Fit of New Keynesian Models" by Del Negro et al. (2006), published on the April 2007 issue of the JBES with comments by L.Christiano, R.Gallant, C.Sims, J.Faust, and L.Killian.

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the structural model. The statistical model of reference for the evaluation of the structural model is derived by the structural model itself. Spanos (1990) points out that the root of the failure of the Cowles Commission approach lies in the little attention paid to the statistical model implicit in the estimated structure. Any identified structure that is estimated without checking that the implied statistical model is an accurate description of the data is bound to fail if the statistical model is not valid. The Spanos critique of the Cowles commission approach lies naturally within the LSE approach to econometric modeling. Such an approach reverses the prominence of the structural model with respect to the reduced form representation. The LSE approach starts its specification and identification procedure with a general dynamic reduced form model. The congruency of such a model cannot be directly assessed against the true DGP, which is unobservable. However, model evaluation is made possible by applying the general principle that congruent models should feature true random residuals; hence, any departure of the vector of residuals from a random normal multivariate distribution should signal a mis-specification. A structural model can be identified and estimated only after a validation procedure based on a battery of tests on the reduced form residuals has been satisfactorily implemented. A just-identified specification does not require any further testing, as its implied reduced form does not impose any further restrictions on the baseline statistical model. The validity of over-identified specification is instead tested by evaluating the validity of the restrictions implicitly imposed on the general reduced form. Interestingly, the lack of statistical identification offers an explanation for the failure of the Cowles Commission models very different from the "great critiques" by Lucas (1976) and Sims (1980), that concentrate on model failure related to structural identification problems.

The structural identification problem for DSGE has recently received some close attention (Canova and Sala, 2006). This paper concentrates on the statistical identification model of DSGE models. We illustrate how the logic of some recently proposed model evaluation tools for DSGE models, based on the comparative evaluation of a DSGE–VAR model with an unrestricted VAR model, resembles closely the logic applied within the Cowles Commission approach in testing for the validity of over-identifying restrictions in structural models. We then show that statistical identification can be achieved by using a Factor Augmented VAR (FAVAR), and we compare the properties of DSGE–VAR and DSGE–FAVAR. We provide an empirical illustration by considering the case of a very simple three-equations DSGE model (Del Negro and Schorfheide, 2004).

## 2. Statistical identification: The original concept

Spanos (1990) considers the case of a simple demand and supply model to show how the reduced form is ignored in the traditional approach. The example is based on the market for commercial loans discussed in Maddala (1988). Most of the widely used estimators allow the derivation of numerical values for the structural parameters without even seeing the statistical models represented by the reduced form. Following this tradition, the estimated (by 2SLS) structural model is a static model that relates the demand for loans to the average prime rate, to the Aaa corporate bond rate and to the industrial production index, while the supply of loans depends on the average prime rate, the three-month bill rate and total bank deposits. The quantity of commercial loans and the average prime rate are considered as endogenous while all other variables are taken as, at least, weakly exogenous variables in the sense of Engle et al. (1983)and no equation for them is explicitly estimated. Given that there are two omitted instruments in each equation, one over-identifying restriction is imposed in both the demand and supply equations. The validity of the restrictions is tested via the Anderson and Rubin (1949) tests, and leads to the rejection of the restrictions at the 5% level in both equations, although in the second equation the restrictions cannot be rejected at the 1% level. This mild evidence against the adopted structural model ignores the fact that estimation of the statistical model, i.e. the reduced form implied by the adopted identifying restrictions, yields a specification for which the underlying statistical assumptions of linearity, homoscedasticity, absence of autocorrelation and normality of residuals are all strongly rejected. On the basis of this evidence the adopted statistical model is not considered as appropriate. An alternative model allowing for a richer dynamic structure (two lags) in the reduced form is then considered. Such dynamic specification is shown to provide a much better statistical model for the data than the static reduced from. Of course, the adopted structural model implies many more over-identifying restrictions than the initial more parsimonious specification. When tested, the validity of these restrictions is overwhelmingly rejected for both the demand and the supply equations. Such evidence leads to the conclusion that the lack of statistical identification of the original model might lead to failure of rejecting the structural model of interest when it is false.

In practice, Cowles Commission models have been abandoned because of their empirical failure and because of the great critiques related to their lack of structural identification, much less emphasis has been posed by the mainstream literature on the problem of statistical identification, with the notable exception of the LSE approach to econometric dynamics (see, Hendry (1995)). Cowles Commission models for policy evaluation have been replaced by Dynamic Stochastic General Equilibrium (DSGE) models.

## 3. The statistical identification of VAR and DSGE models

The general linear (or linearized around equilibrium) DSGE model takes the following form (see Sims (2002)):

$$\mathbf{\Gamma}_0 \mathbf{Z}_t = \mathbf{\Gamma}_1 \mathbf{Z}_{t-1} + C + \Psi \epsilon_t + \Pi \eta_t \tag{1}$$

where $C$ is a vector of constants, $\epsilon_t$ is an exogenously evolving random disturbance, $\eta_t$ is a vector of expectations errors, $(E_t(\eta_{t+1}) = \mathbf{0})$, not given exogenously but to be treated as part of the model solution. The forcing processes here are the elements of the vector $\epsilon_t$, this typically contains processes like Total Factor Productivity or policy variables that are not determined by an optimization process. Policy variables set by optimization, typically included $\mathbf{Z}_t$, are naturally endogenous as optimal policy requires some response to current and expected developments of the economy. Expectations at time t for some of the variables of the systems at time $t + 1$ are also included in the vector $\mathbf{Z}_t$, whenever the model is forward looking. Model like (1) can be solved using standard numerical techniques (see, for example, Sims (2002)), and the solution can be expressed as:

$$\mathbf{Z}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{Z}_{t-1} + \mathbf{R}\epsilon_t \tag{2}$$

where the matrices $\mathbf{A}_0, \mathbf{A}_1$, and $\mathbf{R}$ contain convolutions of the underlying model structural parameters. Consider the simple case in which all variables in the DSGE are observable and the number of structural shocks in $\epsilon_t$ is exactly equal to the number of variables in $\mathbf{Z}_t$. In this case VAR are natural specifications for the data, therefore the estimated reduced form in modern macroeconometrics is:

$$\mathbf{Z}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{Z}_{t-1} + \mathbf{u}_t. \tag{3}$$

Within this framework a new role for empirical analysis based on reduced form models emerges, that is to provide evidence on the stylized facts to be matched by the theoretical model