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Design sensitivity analysis and optimization of frequency averaged input power using the residue theorem

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ABSTRACT

The residue theorem is employed in the design optimization of the frequency averaged power injected into a linear second-order system. The dynamic behavior of a vibrating structure is obtained with the finite element method and the frequency averaged input power into the structure is adopted as an objective function. The design sensitivity with respect to each structural element is computed by the adjoint variable method with consideration for the residue theorem. The proposed method highly enhances the computational efficiency of the optimization and the design of the optimization leads to substantial reduction of the radiated sound power from the structure.

1. Introduction

Computer aided engineering (CAE) is widely used in the design optimization of vibro-acoustic models. Commonly, a physical quantity, such as sound pressure level [1–6] or sound power [7–12], at a certain frequency is used as an objective function. However, the eventual design realization always exhibits slightly different resonances than the nominal model. Hence, design engineers prefer to consider a certain frequency band rather than a single frequency to obtain a more robust design. Numerical quadrature rules, such as a rectangular rule and a trapezoidal rule, are commonly used for the evaluation of the frequency average over a band. Their computational process is straightforward regardless of the complexity of the objective function. However, a large number of function evaluations at individual frequencies are required to get the accurate average value within the band, and the computational cost of such a strategy increases as the bandwidth becomes larger. Furthermore, the frequency resolution should be high enough in order to capture the oscillatory behavior of a complex response function. When it comes to the design optimization, this process is performed for every design update, thereby resulting in an expensive computational cost to reach an optimum point. To avoid such a cumbersome process, the strategy proposed in recent works by the authors [13,14] using the residue theorem is applied here. They introduce weighting functions in order to filter a certain frequency band and evaluate the system response at a few complex frequencies rather than integrating over real frequencies. Therefore, the strategy allows an efficient process for wide band optimum design of vibro-acoustic problems with a consequent reduction of the computation time in the objective function evaluation.

In vibro-acoustic problems, the vibrating structure is the source of radiated sound. For the reduction of vibration on the time-harmonic oscillated structure, input power called as dynamic compliance has been widely used as an objective function in the design optimization [15–19]. The input power minimization maximizes the input impedance at the excitation point so that the global vibration of the structure is reduced. The radiated sound does not scale proportionally with input power given by the different radiation efficiency of various structural modes; however, minimizing the input power indirectly leads to a substantial reduction of the radiated sound power [19]. Since general mechanical structures are much stiffer than air, the vibration of the structure is assumed not to be affected by the fluid. Therefore the sound power level reduction can be achieved by simply considering the structural dynamic behavior in the optimization process. The input power does not require an acoustic modeling and complex design sensitivity formulations such like used in sound power design optimization, thereby leading to a simpler and more efficient optimization process.

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In this paper, the frequency averaged input power into a hollow cube built of plates is computed by using the residue theorem, and the residue formulation is applied in the optimization process to reduce the radiated sound power of the vibrating cube at the considered frequency band. This novel approach applying the residue scheme in the optimization strategy has never been used. The computational efficiency is highly increased compared to the conventional vibro-acoustic design optimization on a wide band frequency response and the design sensitivity is derived quite straightforwardly so that it can be easily applied to most of the design parameters in the vibro-acoustic problem. The finite element method (FEM) [20,21] is used to investigate the dynamic behavior of a vibrating cube, and the thickness of a group of shell elements is considered as a sizing design variable. For the evaluation of the design sensitivity, the adjoint variable method (AVM) [22] is used due to its computational efficiency. Through the combination of the residue theorem and the adjoint design sensitivity, the computational efficiency is highly increased in the optimization of a wide band frequency average of a vibrating structure comprising large number of design variables. Different types of weighting functions for the frequency averaged input power are considered as objective functions and their results are compared with each other. To validate the noise reduction from the minimum input power model, radiated sound powers of the optimum designs are computed using the wave based method (WBM) [23–25]. The results show that the proposed approach can be a good solution to reduce the radiated sound from vibrating structures over frequency bands.

2. Structural FEM

The design sensitivity is defined by the design derivative of a governing equation. It is essential to understand the governing equation and the numerical prediction scheme to derive the design sensitivity formulation, because the variational equation used in the numerical modeling stage is reused in the design sensitivity formulation. This section presents a brief introduction to the variational formulation and its FEM discretization for a generic structural dynamic problem. The variational equation for a structural dynamic model under the time-harmonic excitation can be obtained as [22]

$$-\omega^2 d(\mathbf{z},\bar{\mathbf{z}}) + j\omega c(\mathbf{z},\bar{\mathbf{z}}) + a(\mathbf{z},\bar{\mathbf{z}}) = \ell(\bar{\mathbf{z}}),\ \forall\bar{\mathbf{z}}\in Z, \tag{1}$$

where ω is the angular frequency, $\bar{\mathbf{z}}$ is the virtual displacement, and Z is the complex space of the kinematically admissible virtual displacements. Each variational term in Eq. (1) is defined as follows

$$d(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S} \rho_s \bar{\mathbf{z}}^{*^{\mathrm{T}}}\mathbf{z}d\Omega^S, \tag{2}$$

$$c(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S} C\bar{\mathbf{z}}^{*^{\mathrm{T}}}\mathbf{z}d\Omega^S, \tag{3}$$

$$a(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S} \boldsymbol{\varepsilon}(\bar{\mathbf{z}}^*)^T\boldsymbol{\sigma}(\mathbf{z})d\Omega^S, \tag{4}$$

$$\ell(\bar{\mathbf{z}}) = \int\int_{\Omega^S} \bar{\mathbf{z}}^{*^{\mathrm{T}}}\mathbf{f}d\Omega^S, \tag{5}$$

where $d(\mathbf{z},\bar{\mathbf{z}})$ is the kinetic sesqui-linear form, $c(\mathbf{z},\bar{\mathbf{z}})$ is the damping sesqui-linear form, $a(\mathbf{z},\bar{\mathbf{z}})$ is the structural sesqui-linear form, $\ell(\bar{\mathbf{z}})$ is the load semi-linear form, Ω^S is the domain of the structure, ρ_s and C are the structural mass density and viscous damping, $\bar{\mathbf{z}}^*$ is the complex conjugate of $\bar{\mathbf{z}}$, and $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are the strain and stress tensor, respectively. The structural-state variable $\mathbf{z}$ of the shell component is defined by

$$\mathbf{z} = [z_1, z_2, z_3, \theta_1, \theta_2]^{\mathrm{T}}. \tag{6}$$

Fig. 1 shows a rectangular shell description with the component of the state variable in Eq. (6). The strain $\boldsymbol{\varepsilon}$ for a thin plate shell element is decomposed into its membrane and bending parts, as shown in Eqs. (7) and (8), respectively,

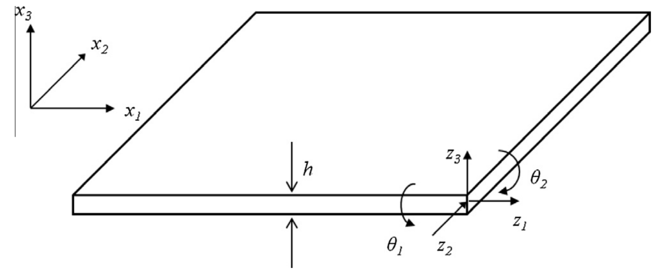

Fig. 1. Description of the state variable on the rectangular shell element with thickness h.

$$\boldsymbol{\varepsilon}^m = \begin{bmatrix} z_{1,1} \\ z_{2,2} \\ z_{1,2}+z_{2,1}\end{bmatrix}, \quad \boldsymbol{\kappa} = \begin{bmatrix} \theta_{1,1} \\ \theta_{2,2} \\ \theta_{1,2}+\theta_{2,1}\end{bmatrix}, \tag{7,8}$$

where $\theta_{1,1}$ and $\theta_{2,2}$ are the bending curvatures in the x_1 and x_2 direction, respectively, and $\theta_{1,2} + \theta_{2,1}$ is the twisting curvature [2]. By using Eqs. (7) and (8), the kinetic and the structural sesqui-linear form for the thin plate shell element can be expressed by considering its thickness h as

$$d(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S} h\rho_s\bar{\mathbf{z}}^{*^{\mathrm{T}}}\mathbf{z}d\Omega^S, \tag{9}$$

$$a(\mathbf{z},\bar{\mathbf{z}}) = \int\int_{\Omega^S}\left[h\boldsymbol{\varepsilon}^m(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{C}\boldsymbol{\varepsilon}^m(\mathbf{z}) + \frac{h^3}{12}\boldsymbol{\kappa}(\bar{\mathbf{z}}^*)^{\mathrm{T}}\mathbf{C}\boldsymbol{\kappa}(\mathbf{z})\right]d\Omega^S, \tag{10}$$

and the strain–stress matrix under the assumption of plane stress is defined as

$$\mathbf{C} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}. \tag{11}$$

A damping loss factor of many mechanisms remains constant within certain frequency ranges and mainly influences the frequency response function in correspondence of its natural frequencies when the damping is low [26]. Therefore the viscous damped dynamic model can be substituted by using the linear hysteretic damping, called as structural damping, with the use of the loss factor.

To make the computation of the input power straightforward, the state variable in the variational Eq. (1) can be conveniently changed into velocity using the relationship $\mathbf{v}(\mathbf{x}) = j\omega\mathbf{z}(\mathbf{x})$. By taking the structural damping effect into account in the variational equation for the structural dynamic system, Eq. (1) can be represented as

$$j\omega d(\mathbf{v},\bar{\mathbf{z}}) + \kappa a(\mathbf{v},\bar{\mathbf{z}}) = \ell(\bar{\mathbf{z}}), \tag{12}$$

where $\kappa = (1 + j\varphi)/j\omega$ and φ is the loss factor. After discretization of the variational equation using FEM and a reduction of the global matrices using kinematic boundary conditions, the variational form of Eq. (12) can be approximated as [20, 21]

$$[j\omega\mathbf{M} + \kappa\mathbf{K}]\{\mathbf{v}\} = \{\mathbf{f}\}, \tag{13}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the global inertia and stiffness matrix, respectively.

3. Frequency averaged input power

The input power of a harmonically excited structural model is defined as

$$P_{in}(\omega) = \frac{1}{2}Re\int\int_{\Omega^S}\mathbf{f}^*\mathbf{v}(\omega)d\Omega^S. \tag{14}$$