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A background decomposition method for domain integration in weak-form meshfree methods

M.R. Hematiyan [a,*], A. Khosravifard [a], G.R. Liu [b]

[a] Department of Mechanical Engineering, Shiraz University, Shiraz 71936, Iran
[b] School of Aerospace Systems, University of Cincinnati, 2851 Woodside Dr, Cincinnati, OH 45221, USA

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ABSTRACT

An efficient technique is presented for evaluation of a domain integral in which the integrand is defined by its values at a discrete set of nodes with highly varying density. The proposed technique uses quadtree and octree techniques for 2D and 3D domains, respectively, so that the background of the integration domain can be divided into a few partitions with different grades of nodal density. The integrals over all partitions are then evaluated and added together to get the value of the whole-domain integral. Some numerical examples are given to show the accuracy and efficiency of the presented method.

1. Introduction

Computation of domain integrals using the numerical values of integrands at some internal and boundary points is necessary in some important computational methods, especially for meshfree methods based on weak formulation [1]. In addition, to solve transient, nonlinear, or nonhomogeneous problems using the boundary element method (BEM) we have to consider some internal points besides boundary nodes. These internal points can also be considered for meshfree evaluation of domain integrals appearing in the BEM formulation of the problem [2–7].

Meshfree methods have achieved remarkable progress in recent years and they are still an active research area [8–10]. The existing meshfree methods can be classified as being based on particles [11,12], strong forms [13–15], weak forms [16–20], or weakened weak forms [21–24]. In the methods based on particles and strong forms, discretization is carried out directly from the governing differential equations. For the weakened weak form methods, there is no need for any domain integration for the construction of equations. However, in the methods based on weak forms, the algebraic equations are generated from domain integrals. Compared to the strong form methods, meshfree methods based on weak forms usually result in solutions with better stability and higher accuracy, and are hence widely used [20].

In general, numerical integration in meshfree methods can be more challenging than that in the conventional finite element method (FEM) [25]. However, in the FEM with polygonal elements [26,27], the integration over elements encounters some difficulties. In a two-level method for integration over polygons [28], the domain of the polygon is mapped onto a regular polygon. Then, the regular polygon is triangulated and the integral over each triangle is evaluated by another mapping. Natarajan et al. [29] presented a one-level method based on Schwarz-Christofel mapping for evaluation of integrals over arbitrary polygonal domains.

The overall performance of the meshfree methods rely deeply on the accuracy and efficiency of the domain integrations. The importance of the domain integration has been widely reported in the studies for the element free Galerkin (EFG) method [16], the radial point interpolation method (RPIM) [30], the reproducing kernel particle method (RKPM) [17], the hp clouds method [31], and the partition of unity (PU) method [32]. In addition, in the local boundary integral equation (LBIE) and the meshfree local Petrov–Galerkin (MLPG) methods, a local Galerkin weak form is used, and thus complicated integrations are also required for the local smaller domains [33].

Using a uniform background mesh [16,19] is currently the most popular technique for the evaluation of domain integrals in weak-form meshfree methods. The nodal integration [34] and the stabilized conforming nodal integration [35,36] methods are other techniques for evaluation of domain integrals in meshfree methods. These methods are based on the construction of Voronoi diagram, and are known to have the so-called temporal instability problems. Racz and Bui [37] proposed another method to get rid

* Corresponding author. Tel.: +98 711 6133149; fax: +98 711 6473511.
E-mail addresses: mo.hematiyan@gmail.com, mhemat@shirazu.ac.ir (M.R. Hematiyan).

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of the difficulties associated with the use of background mesh. They used specific mapping techniques to map a complex integration domain to simple domains. Another method, which is widely used for evaluation of domain integrals in meshfree methods, is the Gaussian quadrature along with a finite element background mesh [1]. In this context, a triangular or tetrahedral finite element mesh is generated using the field nodes. The vertices of the triangles or tetrahedrons are the same as the nodal points and the elements are used for evaluation of the domain integrals. Such elements are generated solely for evaluation of the domain integrals, and the interpolation of the primary variable is performed with the standard meshfree interpolation techniques. This integration technique usually results in more accurate results; however, this technique makes the method being not "truly" meshfree. A helpful review and discussion of frequently used integration techniques in meshfree methods can be found in [38,39].

In some problems, the field variable, e.g., temperature, displacement, or stress may have high local variations in some parts of the region. For such problems, two approaches are usually utilized to obtain accurate solutions. The first approach is to use a dense grid of nodes in the region with high variation of the field variable [40–42]. The second approach is based on the enrichment of the basis functions approximating the field variables [43–46]. The enrichment methods are widely used in meshfree methods for fracture analysis. These methods can be employed with or without asymptotic enrichment [47,48].

Hematiyan [49,50] presented a method based on a simple Cartesian transformation for evaluation of 2D and 3D domain integrals in the BEM. Khosravifard and Hematiyan [51] extended the Cartesian transformation method (CTM) for evaluation of domain integrals in meshfree methods. The CTM is a truly meshfree technique and is more efficient than the integration methods based on a background mesh [51]. However, the CTM is not efficient enough for evaluation of an integral over a domain in which the density of nodes has a severe variation over the domain.

In this paper, a novel method for evaluation of domain integrals with non-uniform distribution of nodes is developed. We assume that the variation characteristics of the integrand are captured by the distribution of the nodes. Therefore, the distribution of the integration points is adjusted to that of the field nodes, i.e. a dense distribution of integration points is automatically selected where the nodes are closer to each other. This method is called background decomposition method (BDM) for convenient reference. Using quadtree and octree partitioning algorithms for 2D and 3D domains, respectively, a square (cube) covering the problem domain is converted into a few partitions with different grades of nodes density. The integral over each partition is separately evaluated. It should be mentioned that there are several variations of the FEM in which the analysis mesh is not conformal to the geometry. The use of quadtree-based strategies for mesh generation and/or integration is also studied in these methods. Some of these methods are generalized FEM [52,53], Cartesian grid FEM [54], extended FEM [55,53], fixed grid FEM [56,57], and immersed FEM [58]. On the other hand, adaptive refinement strategies in meshfree methods may be based on quadtree and octree partitioning algorithms [59].

The BDM is especially useful for the evaluation of domain integrals in problems where the density of nodes varies severely in the problem domain. Problems associated with fracture mechanics, stress concentration, application of point or line loads are some examples of such cases. In such problems, a more compact distribution of nodes is selected in regions, where a problem variable has a sharp slope or even is singular. We will demonstrate through some examples that the BDM can accurately and efficiently evaluate domain integrals in domains with irregularly and non-uniformly distributed nodes. The accuracy of the presented method is also compared with the standard integration technique that uses the well-established Gaussian quadrature with triangular finite element background mesh [1]. This technique is specially selected for comparison purposes because it is regarded as one of the most accurate methods for evaluation of domain integrals in the meshfree weak-form methods. Through the numerical examples, it will be assessed that the BDM is as accurate as the Gaussian quadrature with finite element (FE) background mesh, while being independent of domain meshing. To be more specific, with the same number of integration points and floating-point operations, the BDM is slightly more accurate than the Gaussian quadrature with FE background mesh. However, the main advantage of the BDM is getting rid of the operations associated with meshing of the problem domain. This issue is especially important in the analysis of problems requiring remeshing of the problem domain. In such problems, addition or removal of the nodes to or from the problem domain poses no difficulty on the BDM procedure. This is in contrast to the integration methods based on FE background mesh, which need to form a new mesh in case of addition or removal of nodes.

2. Description of the problem

Consider that the following domain integral is to be computed numerically

$$I = \int_{\Omega} g(\mathbf{x}) d\Omega. \tag{1}$$

where Ω is the integration domain and the integrand g is a regular function. It is assumed that the function g is represented by its values at M discrete nodes. The nodes may be distributed over the domain with an approximately uniform density as shown in Fig. 1(a) or with a non-uniform distribution as shown in Fig 1(b) and (c).

In the present work, we present a meshfree integration technique for the evaluation of the domain integral in Eq. (1). In this method, the local geometry of the problem domain, as well as the nodal distribution of the meshfree method are taken into account for the selection of the integration points. This is to ensure that each part of the domain receives adequate density of integration points for the desired accuracy in the evaluation of the domain integral.

3. Meshfree interpolation methods

Meshfree interpolation methods are used to interpolate a set of discrete data scattered through a domain and on its boundary using local nodes without considering their connectivity. There are three different types of meshfree interpolation techniques, i.e., finite integral representation, finite series representation, and finite differential representation methods [1]. Finite series representation methods are more popular and are briefly described in this section. These methods are used to obtain an approximate function, based on the nodal values of a discrete set of data, i.e.

$$g(x,y,z) = \sum_{j=1}^{m} \phi_j(x,y,z) g_j = \mathbf{\Phi}^{\mathrm{T}}\mathbf{g}, \tag{2}$$

where the vector $\mathbf{g}$ contains the nodal values of the function g at the m local nodal points and $\mathbf{\Phi}$ is the shape function vector of the meshfree interpolation method. In the case of local interpolation methods, m is less than the total number of nodes. By the direct differentiation of Eq. (2), the derivative of the function can also be found to the desired order. For instance, the first-order derivative of g can be written as: