Available online at www.sciencedirect.com
ScienceDirect
Journal of Econometrics 141 (2007) 1245–1280
JOURNAL OF Econometrics
www.elsevier.com/locate/jeconom

Closed-form likelihood approximation and estimation of jump-diffusions with an application to the realignment risk of the Chinese Yuan

Jialin Yu*

Department of Finance and Economics, Columbia University, 421 Uris Hall, 3022 Broadway, New York, NY 10027, USA

Available online 27 March 2007

Abstract

This paper provides closed-form likelihood approximations for multivariate jump-diffusion processes widely used in finance. For a fixed order of approximation, the maximum-likelihood estimator (MLE) computed from this approximate likelihood achieves the asymptotic efficiency of the true yet *uncomputable* MLE as the sampling interval shrinks. This method is used to uncover the realignment probability of the Chinese Yuan. Since February 2002, the market-implied realignment intensity has increased fivefold. The term structure of the *forward realignment rate*, which completely characterizes future realignment probabilities, is hump-shaped and peaks at mid-2004. The realignment probability responds quickly to economic news releases and government interventions.

JEL classification: C13; C22; C32; G15

Keywords: Maximum likelihood estimation; Jump diffusion; Discrete sampling; Chinese Yuan; Currency realignment

1. Introduction

Jump-diffusions are very useful for modeling various economic phenomena such as currency crises, financial market crashes, defaults etc. There are now substantial evidence

*Tel.: +1 212 854 9140; fax: +1 212 316 9180.
E-mail address: jy2167@columbia.edu.

doi:10.1016/j.jeconom.2007.02.003

of jumps in financial markets. See for example Andersen et al. (2006), Barndorff-Nielsen and Shephard (2004) and Huang and Tauchen (2005) on jumps in modeling and forecasting return volatility, Andersen et al. (2002) and Eraker et al. (2003) on jumps in stock market, Piazzesi (2005) and Johannes (2004) on bond market, Bates (1996) on currency market, Duffie and Singleton (2003) on credit risk. From a statistical point of view, jump-diffusion nests diffusion as a special case. Being a more general model, jump-diffusion can approximate the true data-generating process better as measured by, for example, Kullback–Leibler information criterion (KLIC) (see White, 1982). As illustrated by Merton (1992), jumps model "Rare Events" that can have a big impact over a short period of time. These rare events can have very different implications from diffusive events. For example, it is known in the derivative pricing theory that, with jumps, the arbitrage pricing argument leading to the Black–Scholes option pricing formula breaks down. Liu et al. (2003) have shown that the risks brought by jumps and stochastic volatility dramatically change an investor's optimal portfolio choice.

To estimate jump-diffusions, likelihood-based methods such as maximum-likelihood estimation are preferred. This optimality is well documented in the statistics literature. However, maximum-likelihood estimation is difficult to implement because the likelihood function is available in closed-form for only a handful of processes. This difficulty can be seen from Sundaresan (2000): "The challenge to the econometricians is to present a framework for estimating such multivariate diffusion processes, which are becoming more and more common in financial economics in recent times. ... The development of estimation procedures for multivariate AJD processes is certainly a very important step toward realizing this hope." Note the method proposed in this paper will apply to both affine jump-diffusion (AJD in the quote) and non-AJD. The difficulty of obtaining closed-form likelihood function has led to likelihood approximation using simulation (Pedersen, 1995, Brandt and Santa-Clara, 2002) or Fourier inversion of the characteristic function (Singleton, 2001; Aït-Sahalia and Yu, 2006). Generalized method of moments estimation (Hansen and Scheinkman, 1995; Kessler and Sørensen, 1999; Duffie and Glynn, 2004; Carrasco et al., 2007) has also been proposed to sidestep likelihood evaluation. Non-parametric and semiparametric estimations have also been proposed for diffusions (Aït-Sahalia, 1996; Bandi and Phillips, 2003; Kristensen, 2004) and for jump-diffusions (Bandi and Nguyen, 2003).

The first step towards a *closed-form* likelihood approximation is taken by Aït-Sahalia (2002) who provides a likelihood expansion for univariate diffusions. Such closed-form approximations are shown to be extremely accurate and are fast to compute by Monte-Carlo studies (see Jensen and Poulsen, 2002). The method was subsequently refined by Bakshi and Ju (2005) and Bakshi et al. (2006) under the same setup of univariate diffusions and was extended to multivariate diffusions by Aït-Sahalia (2006) and univariate Levy-driven processes by Schaumburg (2001).

Building on this closed-form approximation approach, this paper provides a closed-form approximation of the likelihood function of multivariate jump-diffusion processes. It extends Aït-Sahalia (2002) and Aït-Sahalia (2006) by constructing an alternative form of leading term that captures the jump behavior. The approximate likelihood function is then solved from Kolmogorov equations. It extends Schaumburg (2001) by relaxing the i.i.d. property inherent in Levy-driven randomness and by addressing multivariate processes. The maximum-likelihood estimators (MLEs) using the approximate likelihood function provided in this paper are shown to achieve the asymptotic efficiency of the true yet