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Quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects when both n and T are large[☆]

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ABSTRACT

This paper investigates the asymptotic properties of quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects, when both the number of individuals n and the number of time periods T are large. We consider the case where T is asymptotically large relative to n, the case where T is asymptotically proportional to n, and the case where n is asymptotically large relative to T. In the case where T is asymptotically large relative to n, the estimators are $\sqrt{nT}$ consistent and asymptotically normal, with the limit distribution centered around 0. When n is asymptotically proportional to T, the estimators are $\sqrt{nT}$ consistent and asymptotically normal, but the limit distribution is not centered around 0; and when n is large relative to T, the estimators are T consistent, and have a degenerate limit distribution. The estimators of the fixed effects are $\sqrt{T}$ consistent and asymptotically normal. We also propose a bias correction for our estimators. We show that when T grows faster than $n^{1/3}$, the correction will asymptotically eliminate the bias and yield a centered confidence interval.

1. Introduction

Spatial econometrics deals with the spatial interactions of economic units in cross-sectional and/or panel data. To capture correlation among cross-sectional units, the spatial autoregressive (SAR) model by Cliff and Ord (1973) has received the most attention in economics. It extends autocorrelation in times series to spatial dimensions, and captures interactions or competition among spatial units. Early development in estimation and testing is summarized in Anselin (1988), Cressie (1993), Kelejian and Robinson (1993), and Anselin and Bera (1998), among others.

Spatial correlation and dynamic settings can be extended to panel data models (Anselin, 1988; Baltagi et al., 2003). Kapoor et al. (2007) provide a rigorous theoretical framework for analysis of spatial panel methods. The model considered for estimation in Kapoor et al. (2007), is a regression panel model with SAR and error components disturbances. Baltagi et al. (2007) consider the testing of spatial and serial dependence in an extended model, where serial correlation on each spatial unit over time, in addition to spatial dependence across spatial units are allowed in the disturbances. These panel models do not incorporate time lagged dependent variables as dynamic structures in the regression equation. By allowing spatial and dynamic features in a regression model, Anselin (2001) distinguishes spatial dynamic models into four categories, namely, "pure space recursive" if only a spatial time lag is included; "time-space recursive" if an individual time lag and a spatial time lag are included; "time-space simultaneous" if an individual time lag and a contemporaneous spatial lag are specified; and "time-space dynamic" if all forms of dependence are included.

In this paper, we shall consider the maximum likelihood (ML) or, more generally, the quasi maximum likelihood (QML) estimation of the spatial dynamic panel data (SDPD) model in the general time-space dynamic category. Because the time-space dynamic category is the general one, our asymptotic analysis and results are applicable to the other three categories as special cases. As a panel model, individual effect (error components) is incorporated in the disturbances. We shall provide a rigorous

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theoretical analysis on the asymptotic properties of the ML estimator (MLE) and the QML estimator (QMLE). The asymptotics will be based on both n, the cross sectional units, and T, the time length, go to infinity, or n being a fixed finite integer, while T goes to infinity. The case with both n and T going to infinity will be the main interest.

As our model includes the dynamic panel data model without spatial dependence as a special case, estimation issues of the dynamic panel data models in the existing econometric literature are relevant. When the time dimension T is fixed, we are likely to encounter the "incidental parameters" problem discussed in Neyman and Scott (1948). This is because the introduction of fixed effects increases the number of parameters to be estimated. In a simple dynamic panel data model with fixed effects, the MLE of the autoregressive coefficient, which is also known as the within group estimator, is biased and inconsistent when n tends to infinity but T is fixed (Nickell, 1981; Hsiao, 1986). To avoid the incidental parameters problem in estimation, alternative estimation methods have been introduced. By taking time differences to eliminate the fixed effects in either the dynamic equation or the construction of instrumental variables (IV), Anderson and Hsiao (1981) show that IV methods can provide consistent estimates. Arellano and Bond (1991) and Arellano and Bover (1995) generalize Anderson and Hsiao (1981) with many more IV moments, by exploring all possible time lag values of the dependent variable in each time period. Blundell and Bond (1998) have considered system estimators, including moments of both levels and first differences in Arellano and Bond (1991) and Arellano and Bover (1995). Bun and Kiviet (2006) derive higher order asymptotic approximation of the finite sample bias for the system estimator under various circumstances, as both N and T are small or moderately large. When T is finite, additional IVs can improve the efficiency of the estimators, even though finite sample biases remain. When both n and T go to infinity, the incidental parameters issue in the MLE becomes less severe as each individual fixed effect can be consistently estimated. However, Hahn and Kuersteiner (2002) and Alvarez and Arellano (2003) have found the existence of asymptotic bias of order $O(1/T)$ in the MLE of the autoregressive parameter when both n and T tend to infinity at a proportional rate. In addition to the MLE, Alvarez and Arellano (2003) also investigate the asymptotic properties of the IV estimators in Arellano and Bond (1991). They have found the presence of asymptotic bias of a similar order to that of the MLE and the IV estimators, due to the presence of many moment conditions. The presence of asymptotic bias is an undesirable feature of these estimates.

Kiviet (1995), Hahn and Kuersteiner (2002), and Bun and Carree (2005) have constructed bias corrected estimators for the dynamic panel data model, by analytically modifying the within estimator. Hahn and Kuersteiner (2002) provide a rigorous asymptotic theory for the within estimator and their bias corrected estimator, when both n and T go to infinity with a same rate. As an alternative to the analytical bias correction, Hahn and Newey (2004) have considered also the Jackknife bias reduction approach.

For the SAR model, Kelejian and Prucha (1998) provide a theoretical foundation for asymptotic analysis for their IV estimator. Lee (2004) analyzes the asymptotic properties of the QMLE. Kapoor et al. (2007) extend their asymptotic analysis of IV and method of moments estimators to a spatial panel model with error components, where T is a fixed finite integer. To the best knowledge of the authors, there is little analytical work done on estimates of spatial dynamic models, when both n and T are large, with the exception of Korniotis (2005). The model considered in Korniotis (2005) is a time-space recursive model in that only individual time lag and spatial time lag are present, but not contemporaneous spatial lag. Fixed effects are included in the model, and this model has an empirical application to US state consumption growth. As a recursive model, the parameters including the fixed effects can be estimated by OLS (within estimator). Korniotis (2005) has also considered a bias adjusted within estimator, which generalizes that in Hahn and Kuersteiner (2002). For the dynamic spatial model considered in this paper, as the contemporaneous spatial lag is presented, the QMLEs of the parameters are nonlinear. Our asymptotic analysis is more complex, but our assumptions are more general. The asymptotics in Hahn and Kuersteiner (2002) is based on the scenario that n and T diverge at a proportional rate. Our asymptotic analysis can cover this scenario and also scenarios that n may go to infinity faster than T, and vice versa. Following the literature on bias correction, we have also considered a bias-adjusted estimator for our QMLE and its asymptotic properties. Monte Carlo experiments are conducted to provide some finite sample properties of the estimators. This paper is theoretic and does not provide an empirical application. But it is interesting to note that the empirical study on interregional trade with a historical panel data on Chinese rice price by Keller and Shiue (2007) allows own time and spatial time lags in addition to a contemporaneous spatial lag in their spatial model.[1]

This paper is organized as follows. In Section 2, we introduce the model, and explain our estimation method, which is a concentrated QML estimation. With the law of large numbers and central limit theorem for our setting developed in the Appendix, Section 3 establishes the consistency and asymptotic distributions of MLE and QMLE. We also propose an analytical bias correction for our estimators. We show that when T grows faster than $n^{1/3}$, this correction will eliminate the bias, and yield a centered confidence interval. Section 4 concludes the paper. Some useful lemmas and proofs are collected in the Appendix.[2]

2. The model and concentrated likelihood function

2.1. The model

The model considered in this paper is

$$Y_{nt} = \lambda_0 W_n Y_{nt} + \gamma_0 Y_{n,t-1} + \rho_0 W_n Y_{n,t-1} + X_{nt}\beta_0 + \mathbf{c}_{n0} + V_{nt},\quad t = 1, 2, \ldots, T, \tag{1}$$

where $Y_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$ and $V_{nt} = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ are $n \times 1$ column vectors and v_{it} is *i.i.d.*. across i and t with zero mean and variance σ_0^2, W_n is an $n \times n$ spatial weights matrix, which is predetermined and generates the spatial dependence between cross sectional units y_{it}, X_{nt} is an $n \times k_x$ matrix of nonstochastic regressors, and $\mathbf{c}_{n0}$ is $n \times 1$ column vector of fixed effects. Therefore, the total number of parameters in this model is equal to the number of individuals n plus the dimension of the common parameters $(\gamma, \rho, \beta', \lambda, \sigma^2)'$, which is $k_x + 4$.

Define $S_n \equiv S_n(\lambda_0) = I_n - \lambda_0 W_n$. Then, presuming S_n is invertible and denoting $A_n = S_n^{-1}(\gamma_0 I_n + \rho_0 W_n)$, (1) can be rewritten as $Y_{nt} = A_n Y_{n,t-1} + S_n^{-1} X_{nt}\beta_0 + S_n^{-1}\mathbf{c}_{n0} + S_n^{-1} V_{nt}$. Assuming that the infinite sums are well-defined, by continuous substitution,

$$\begin{aligned} Y_{nt} &= \sum_{h=0}^{\infty} A_n^h S_n^{-1}(\mathbf{c}_{n0} + X_{n,t-h}\beta_0 + V_{n,t-h}) \\ &= \mu_n + \mathcal{X}_{nt}\beta_0 + U_{nt}, \end{aligned} \tag{2}$$

where $\mu_n \equiv \sum_{h=0}^{\infty} A_n^h S_n^{-1}\mathbf{c}_{n0}$, $\mathcal{X}_{nt} \equiv \sum_{h=0}^{\infty} A_n^h S_n^{-1} X_{n,t-h}$, and $U_{nt} \equiv \sum_{h=0}^{\infty} A_n^h S_n^{-1} V_{n,t-h}$.

[1] However, error components have not been considered in their empirical models and no theoretic properties of the estimates are investigated in the paper.

[2] Due to space limitation, at the request of the editor and referees, some of the proofs have been condensed and removed. The detailed proofs and intermediate steps in some derivations can be found in the working paper version of this paper. The working paper under the same title is available on the web site: http://economics.sbs.ohio-state.edu/lee/.