Computers and Structures 139 (2014) 28–32

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Component mode synthesis with subspace iterations for controlled accuracy of frequency and mode shape solutions

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ARTICLE INFO

Article history:
Received 31 January 2014
Accepted 16 March 2014
Available online 9 May 2014

Keywords:
Frequencies and mode shapes
Component mode synthesis
Craig–Bampton scheme
Bathe subspace iteration method
Control of errors

ABSTRACT

The objective in this paper is to present an approach to improve component mode synthesis solutions using subspace iterations to obtain frequency and mode shape predictions of controlled accuracy. In traditional component mode synthesis analyses, the calculated frequencies and mode shapes are approximations of the exact frequencies and mode shapes of the finite element model, the error is unknown, may be large, and is usually not assessed. In the approach given here, the error is assessed and can be reduced to the desired level. The Craig–Bampton component mode synthesis is used, but the solution approach is also directly applicable to any other component mode synthesis scheme. Some example solutions are given to illustrate the use and the effectiveness of the solution approach.

1. Introduction

Component mode synthesis solution techniques [1,2] are widely used in finite element analyses. The solution approach was proposed long ago by Hurty [3], but when seen as a domain decomposition technique can in fact be traced back to much earlier work, see Ref. [4]. These methods can be effective for the solution of frequencies and mode shapes when complex structures are analyzed, as, for example, in the airplane industries. The representation of the complete structure is based on using large substructures. These individual structures, or components, are typically first analyzed in detail for their frequencies and mode shapes, and dynamic response, by different analysis groups. Then the complete structure is considered as an assemblage of the components, which can lead to a very large finite element system. However, usually only the lowest p frequencies and corresponding mode shapes of the complete finite element system with n degrees of freedom are needed, where $p \ll n$. The basic approach of a component mode synthesis analysis is to use the mode shape solutions of the individual components, i.e. substructures, to obtain approximations to the exact p frequencies and mode shapes of the complete model [2]. An important such procedure used is the Craig–Bampton method [5].

Since the component mode synthesis procedures only give approximate solutions to the exact eigenvalues and vectors (frequencies and mode shapes) of the complete structural model, it can be important to have some error measure, see Refs. [4,6] and the many references therein, to ensure the reliability of a response prediction. The scheme should give an approximation to the error in the solution when compared to the exact solution, which of course is unknown. Considering that the calculated eigenvalues are always larger than the exact values [2], the error measure should ideally give a proven upper bound to the exact error, be close to the exact error, and should be inexpensive to compute. Also, if the error is too large, a simple scheme to reduce the error should be available. This approach is valuable because in today's finite element analysis practice, a strong emphasis should be on the reliability of the numerical solutions rather than merely on computational effectiveness. The reliability should indeed be a requirement in the solution of the required frequencies and mode shapes, also because large systems of finite element equations can now be solved very accurately using the Lanczos transformation method and the Bathe subspace iteration technique [2].

As is well known, component mode synthesis solutions are closely related to Rayleigh–Ritz analysis and hence also to a subspace iteration. It follows therefore that the error measure used in the Bathe subspace iteration method may also be attractive for use in a component mode synthesis solution [2,7–9].

In recent work, Yin et al. [10] presented automated multilevel substructuring (AMLS) techniques using also the Bathe subspace iteration method with an error indicator. However, the emphasis in the following sections is on using a proven error bound and a simple scheme to decrease the error, when so wanted, monotonically to

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http://dx.doi.org/10.1016/j.compstruc.2014.03.003
0045-7949/

the desired value. This approach is used to obtain reliable solution results.

In the following sections, we review the Craig–Bampton component mode synthesis analysis and show how, in a natural further step, the Bathe subspace iteration method can be used to measure and reduce the error. While we focus on the Craig–Bampton technique, other methods can also be used with the given approach and error measure. Some illustrative example solutions are included to show the applicability of the solution approach.

2. The Craig–Bampton method

We consider the eigenvalue problem

$$\mathbf{K}\boldsymbol{\varphi} = \lambda\mathbf{M}\boldsymbol{\varphi} \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the symmetric stiffness and mass matrices of the complete finite element system of n equations. We seek the smallest p eigenvalues and corresponding eigenvectors, with $p \ll n$. Without loss of generality, we assume that the stiffness and mass matrices $\mathbf{K}$ and $\mathbf{M}$ are positive definite, and hence the eigenvalues we seek are

$$0 < \lambda_1 \leqslant \lambda_2 \cdots \leqslant \lambda_{p-1} \leqslant \lambda_p \tag{2}$$

with the corresponding $\mathbf{M}$-orthonormal eigenvectors $\boldsymbol{\varphi}_1, \ldots, \boldsymbol{\varphi}_p$. For an unsupported structure we would simply apply a shift [2].

The Craig–Bampton method is a well-known component mode synthesis technique, and was designed when the calculation of the exact eigenvalues and vectors (or very close approximations thereof) of very large finite element models was technologically out of reach, or at least computationally very expensive to perform. The method is also naturally optimizing the workflow by using, in the solution of Eq. (1), the already calculated mode shapes of the components, i.e. of the substructures, that were already considered by different analysis groups [1–5].

Consider a generic finite element model as shown in Fig. 1. For the development of the governing equations, the dynamic equilibrium equations are partitioned as follows

$$\begin{bmatrix} \mathbf{M}_{bb} & \mathbf{M}_{bi} \\ \mathbf{M}_{ib} & \mathbf{M}_{ii} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{u}}_i \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{u}_b \\ \mathbf{u}_i \end{bmatrix} = \begin{bmatrix} \mathbf{R}_b \\ \mathbf{R}_i \end{bmatrix} \tag{3}$$

where the entries in $\mathbf{u}_b$ refer to the r degrees of freedom of the boundary nodes and the entries in $\mathbf{u}_i$ correspond to the $(n-r)$ degrees of freedom of the interior (non-boundary) nodes. In the Craig–Bampton procedure the following transformation is used [5]

$$\begin{bmatrix} \mathbf{u}_b \\ \mathbf{u}_i \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\Phi}_c & \boldsymbol{\Phi}_n \end{bmatrix} \begin{bmatrix} \mathbf{u}_b \\ \mathbf{q} \end{bmatrix} \tag{4}$$

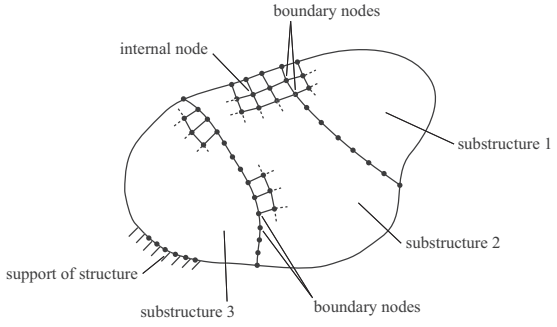

Fig. 1. Generic finite element model using substructuring; the boundary nodes are the nodes shared by substructures.

where $\mathbf{I}$ is the identity matrix, $\boldsymbol{\Phi}_n$ corresponds to s normal modes (corresponding to the smallest eigenvalues) with all boundary degrees of freedom fixed, usually $s \ll n$, $\boldsymbol{\Phi}_c$ is given as

$$\boldsymbol{\Phi}_c = -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} \tag{5}$$

and $\mathbf{q}$ represents generalized displacements. The columns in $\boldsymbol{\Phi}_c$ represent static constraint modes, that is, the jth column represents the static displacements at all interior nodes when all boundary degrees of freedom are fixed except the jth boundary degree of freedom (corresponding to that column) is set to unity.

Hence the procedure uses the transformation matrix

$$\boldsymbol{\Psi} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \boldsymbol{\Phi}_c & \boldsymbol{\Phi}_n \end{bmatrix} \tag{6}$$

where $\boldsymbol{\Psi}$ is of dimension $n \times (r+s)$. Calculating as in a Rayleigh–Ritz solution [2]

$$\mathbf{K}_r = \boldsymbol{\Psi}^T\mathbf{K}\boldsymbol{\Psi} \tag{7}$$

$$\mathbf{M}_r = \boldsymbol{\Psi}^T\mathbf{M}\boldsymbol{\Psi} \tag{8}$$

the reduced eigenvalue problem is solved

$$\mathbf{K}_r\mathbf{X} = \mathbf{M}_r\mathbf{X}\boldsymbol{\rho} \tag{9}$$

The eigenvalue approximations are given in $\boldsymbol{\rho}$ and the eigenvector approximations are listed in $\overline{\boldsymbol{\Phi}}$ as

$$\overline{\boldsymbol{\Phi}} = \boldsymbol{\Psi}\mathbf{X} \tag{10}$$

Various modifications of this basic technique are of course possible, but the essence of all of the related procedures are the steps in Eqs. (7)–(10), with different transformation matrices [1–4].

3. The subspace iteration method

Here too we consider the eigenvalue problem in Eq. (1) and seek the solution of the p lowest eigenvalues and corresponding eigenvectors.

The subspace iteration procedure was developed to simultaneously solve for the required eigenpairs $(\lambda_i, \boldsymbol{\varphi}_i)$ accurately, see Refs. [2,7–9], and has been used abundantly in engineering and the sciences. The basic equations of the Bathe subspace iteration method are:

Pick q starting iteration vectors, $q > p$, to estabish $\mathbf{X}_0$ and then iterate with $k = 1, 2, 3, \ldots$

$$\mathbf{K}\overline{\mathbf{X}}_k = \mathbf{M}\mathbf{X}_{k-1} \tag{11}$$

$$\mathbf{K}_k = \overline{\mathbf{X}}_k^T\mathbf{K}\overline{\mathbf{X}}_k \tag{12}$$

$$\mathbf{M}_k = \overline{\mathbf{X}}_k^T\mathbf{M}\overline{\mathbf{X}}_k \tag{13}$$

$$\mathbf{K}_k\mathbf{Q}_k = \mathbf{M}_k\mathbf{Q}_k\boldsymbol{\Lambda}_k \tag{14}$$

$$\mathbf{X}_k = \overline{\mathbf{X}}_k\mathbf{Q}_k \tag{15}$$

until the following convergence tolerance is passed by all eigenvalues in $\boldsymbol{\Lambda}_k$ to be calculated [2,9]. Since [2,11]

$$\min_{\substack{i \\ \lambda_i \neq 0}} \left| \frac{\lambda_i - \lambda_i^{(k)}}{\lambda_i} \right| \leqslant \left[1 - \frac{\left(\lambda_i^{(k)}\right)^2}{\left(\mathbf{q}_i^{(k)}\right)^T\mathbf{q}_i^{(k)}} \right]^{1/2} \tag{16}$$

the convergence tolerance to be reached is