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Maximum likelihood estimation and inference methods for the covariance stationary panel AR(1)/unit root model

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A R T I C L E I N F O

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This paper considers Maximum Likelihood (ML) based estimation and inference procedures for linear dynamic panel data models with fixed effects.

The paper first studies the asymptotic properties of MaCurdy's [MaCurdy, T., 1982. The use of time series processes to model the time structure of earnings in a longitudinal data analysis. Journal of Econometrics 18, 83–114] First Difference Maximum Likelihood (FDML) estimator for the covariance stationary panel AR(1)/unit root model with fixed effects, viz. $y_{i,t} = \rho y_{i,t-1} + (1-\rho)\mu_i + \varepsilon_{i,t}$, under a variety of asymptotic plans. Subsequently, the paper shows through Monte Carlo simulations for panels of various dimensions the favourable finite sample properties of the FDMLE for ρ as compared to those of a number of alternative fixed effects ML estimators for ρ under covariance stationarity and normality of the data. The paper also discusses panel unit root test procedures that are based on the FDMLE. A Monte Carlo study conducted for one version of these tests reveals that it has very good size and power properties in comparison with alternative panel unit root tests.

1. Introduction

In this paper we discuss various Maximum Likelihood (ML) based estimation and inference procedures for the covariance stationary panel AR(1)/unit root model with fixed effects (FE). We study and compare the properties of several estimators for the autoregressive parameter, ρ, under various asymptotic plans and/or for panels of various dimensions. We also propose a new ML based panel unit root (UR) test and compare it with various existing panel UR tests in a Monte Carlo study.

In the dynamic panel data literature, broadly speaking, two classes of estimators are considered: GMM (IV) estimators and ML estimators. There is now a sizeable literature on GMM estimation of the panel AR(1) model, see e.g. Ahn and Schmidt (1995, 1997) and Arellano (2003). The Generalized Method of Moments owes much of its popularity to its flexibility: one can add or drop moment conditions depending on whether or not specific assumptions about the model are likely to be satisfied by the data. In particular, GMM can be used in the presence of heterogeneous data. For instance, the GMM estimator due to Arellano and Bond (1991) allows for both time-series and cross-sectional heteroskedasticity. However, Monte Carlo studies have revealed that GMM estimators have poor finite sample properties in some cases. For instance, when the value of ρ is close to unity, the Arellano and Bond estimator suffers from a weak instrument problem, see e.g. Blundell and Bond (1998). Moreover, when the number of moment conditions is large relative to the number of observations, e.g. when the number of lags used to form instruments is large, the bias of the Arellano and Bond estimator becomes quite severe, see e.g. Bun and Kiviet (2006).

The other major estimation method, i.e., the ML method, is generally not regarded as a viable alternative to GMM in the case of dynamic panel data models with fixed effects, because it is widely believed that fixed effects ML estimators are inconsistent due to the incidental parameters problem (cf. Neyman and Scott (1948)). The latter belief is probably based on the papers by Kiefer (1980) and Nickell (1981). Nickell has shown that the standard FEML estimator for the panel AR(1) model with arbitrary initial conditions, i.e., the Within Groups (WG) or Least Squares Dummy Variables estimator, is inconsistent when the cross-sectional dimension of the panel, N, tends to infinity whereas the time dimension of the panel, T, is fixed, while Kiefer (1980) has argued that the standard FEML estimator for the covariance matrix

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of the possibly autocorrelated errors of an otherwise static panel regression model is inconsistent when T is fixed.[1]

The assessment of the usefulness of the ML method for estimating dynamic panel data models is not so bleak if one looks further. MaCurdy (1981, 1982) argued that in a situation where T can be treated as fixed, the ML method yields consistent estimators for covariance stationary panel ARMA models with fixed effects when it is applied to first differences of the data. The resulting First Difference ML (FDML) estimators for the AR and MA parameters are still consistent under cross-sectional heteroskedasticity of the errors. More recently, Hsiao et al. (2002) and Kruiniger (2001) have independently shown that the panel AR(1) model with fixed effects and arbitrary initial conditions can be consistently estimated by the ML method, viz. the Restricted FEML (RFEML) estimator, if the differences between the initial observations and the individual effects or, equivalently, the differenced data satisfy a very mild condition, i.e., finite second (or $(2+\delta)$th) moments.[2,3]

In this paper we aim to extend the asymptotic results of MaCurdy (1981, 1982) in two ways. First we investigate further the large N, fixed T asymptotic properties of the FDMLE for the covariance stationary panel AR(1)/UR model giving special attention to the unit root case and the question of efficiency. Among other things we show that the FDMLE does not attain the generalized Cramér–Rao lower bound for estimators for the covariance stationary panel AR(1) model with fixed effects when $N \rightarrow \infty$ and T is fixed. Next, we examine the asymptotic properties of the FDMLE when T cannot be considered fixed. The results of the study provide further insight into the usefulness of the FDMLE as compared to other fixed effects ML estimators for the panel AR(1) model and permit the formulation of new powerful unit root test procedures.

Traditionally, the large N, fixed T asymptotic properties were considered the most relevant asymptotic properties of panel data estimators since the panel data sets used in econometric studies typically had a short time dimension. Because of the increasing availability of panel data sets that have a relatively long time dimension, e.g. the Penn World tables, some attention has recently been given to the properties of estimators under various alternative asymptotic plans in which T grows large, see also Phillips and Moon (1999). For instance, Alvarez and Arellano (2003) and Hahn and Kuersteiner (2002) have derived the asymptotic distribution of the WG estimator under large T, arbitrary N asymptotics and diagonal path asymptotics, respectively. These papers found that if the data are covariance stationary and $0 < \lim(N/T) < \infty$, then the WG estimator has a bias term in its asymptotic distribution. Hahn and Kuersteiner have also developed a bias-corrected version of the WG estimator using the formula for its diagonal path asymptotic bias under covariance stationarity of the data.

In this paper we derive the large T, arbitrary N asymptotic properties of the FDMLE and the standard FEMLE for the covariance stationary panel AR(1) model. We find that under normality and covariance stationarity of the data, the FDMLE for ρ is asymptotically equivalent to the WG estimator when $T \rightarrow \infty$. But in contrast to the WG estimator, the FDMLE for ρ does not exhibit a bias term in its large T, arbitrary N asymptotic distribution unless the assumption of covariance stationarity is not satisfied by the data. Under large T, arbitrary N asymptotics the FDMLE is also asymptotically equivalent to the standard FEMLE for the covariance stationary panel AR(1) model. However, the latter estimator is inconsistent for fixed T and, like the WG estimator, asymptotically biased under large T, arbitrary N asymptotics.

We also show that in the unit root case both under large N, fixed T asymptotics and under joint N, T asymptotics the FDMLE for ρ has a normal limiting distribution. These findings immediately suggest a simple Wald-type panel UR test. The results of Monte Carlo experiments for panels of various dimensions indicate that our FDMLE based UR test has higher power against stationary ($|\rho| < 1$) alternatives than a number of well-known panel UR test from the literature including the test of Harris and Tzavalis (1999) which is based on the WG estimator, and a test of Levin et al. (2002). We also discuss whether and how our panel UR testing procedure should be modified to allow for possibly heterogeneous AR(p) and/or MA(q) dependence, drift or trend parameters, and for non-Gaussian or heterogeneously distributed errors. However, our panel UR test cannot (easily) be modified to allow for unknown structural breaks or general cross-sectional dependence. Recent surveys of the literature on panel UR tests are provided by Bond et al. (2005) and Breitung and Pesaran (2008).

The paper is organised as follows. Section 2 examines the asymptotic properties of the FDMLE for the panel AR(1)/UR model under a variety of asymptotic plans and discusses how the FDML framework can be used to conduct panel UR tests. Section 2 also investigates the asymptotic properties of the standard FEMLE for the covariance stationary panel AR(1) model. Section 3 contains two Monte Carlo studies. First it compares the power and size properties of various panel UR tests including FDMLE based UR tests. Subsequently, it compares the finite sample properties of the FDMLE for ρ with various other fixed effects ML estimators for the covariance stationary panel AR(1) model including two bias-corrected WG estimators. Section 4 concludes the paper. The proofs are collected in the Appendix.

2. ML estimation of the panel AR(1)/UR model with fixed effects

Consider the following panel AR(1) model with individual effects:

$$y_{i,t} = \rho y_{i,t-1} + (1-\rho)\mu_i + \varepsilon_{i,t}, \qquad (1)$$
$$\eta_i = (1-\rho)\mu_i,$$
$$\varepsilon_{i,t}|\eta_i, y_{i,1} \sim N(0,\sigma^2) \quad i.i.d., \quad i = 1,\dots,N, t = 2,\dots,T,$$

where i indicates the individual unit and t indicates the time period. Note that when $\rho = 1$ the individual effects, the μ_i, drop out from the model.

Below we consider various fixed effects (FE) estimators for the covariance stationary panel AR(1) model with possibly a unit root (UR). In the FE version of this model the individual effects are left completely unrestricted. Furthermore, the FE estimators for this model only exploit first differences of the data.

The following assumptions imply that $\{y_{i,t}\}$ is covariance stationary:

$$|\rho| < 1, \quad \text{and}$$
$$(y_{i,1} - \mu_i)|\mu_i \sim N\left(0, \frac{\sigma^2}{1-\rho^2}\right) \quad (i.i.d.), \quad i = 1,\dots,N.$$

[1] Nickell derived a formula for the asymptotic bias of the WG estimator assuming covariance stationarity.

[2] Hsiao et al. called the estimator the Transformed ML estimator rather than the Restricted FEML estimator.

[3] The standard FEML estimators treat the individual effects as N different parameters. On the other hand, the RFEMLE and the related FDMLE assume that the differences between the initial conditions and the individual effects are random variables with a common mean and a common variance parameter. This assumption allows one to formulate a likelihood function for the first differences of the data that is free of incidental parameters. The RFEMLE is equal to the FDMLE when stationarity is imposed on (the common variance parameter of) the initial conditions.