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Testing the parametric form of the volatility in continuous time diffusion models—a stochastic process approach

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Abstract

We present new tests for the form of the volatility function which are based on stochastic processes of the integrated volatility. We prove weak convergence of these processes to centered processes whose conditional distributions are Gaussian. In the case of testing for a constant volatility the limiting process are standard Brownian bridges. As a consequence an asymptotic distribution free test and bootstrap tests (for testing of a general parametric form) can easily be implemented. It is demonstrated that the new tests are more than the currently available procedures. The new approach is also demonstrated by means of a simulation study.

JEL classification: C12

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1. Introduction

Modeling the dynamics of interest rates, stock prices exchange rates is an important problem in mathematical finance and since the seminar papers of Black and Scholes (1973) and Merton (1973) many theoretical models have been developed for this purpose. Most of these models are continuous time stochastic processes, because information arrives at financial markets in continuous time (see Merton, 1990). A commonly used class of processes in mathematical finance for representing asset prices are Itô diffusions defined as a solution of the stochastic differential equation

$$dX_t = b(t, X_t)\,dt + \sigma(t, X_t)\,dW_t, \tag{1}$$

where $(W_t)_t$ is a standard Brownian motion and b and σ denote the drift and volatility, respectively. Various models have been proposed in the literature for the different types of options (see e.g. Black and Scholes, 1973; Vasicek, 1977; Cox et al., 1985; Karatzas, 1988; Constantinides, 1992; Duffie and Harrison, 1993, among many others). For a reasonable pricing of derivative assets in the context of such models a correct specification of the volatility is required and good estimates of this quantity are needed. For example, the pricing of

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European call options crucially depends on the functional form of the volatility (see Black and Scholes, 1973) and the same is true for other types of options (see e.g. Duffie and Harrison, 1993; Karatzas, 1988, among many others).

A (correct) specification of a parametric form for the volatility has the advantage that the problem of its estimation is reduced to the determination of a low-dimensional parameter. On the other hand, a misspecification of drift or variance in the diffusion model (1) may lead to an inadequate data analysis and to serious errors in the pricing of derivative assets. Therefore, several authors propose to check the postulated model by means of a goodness-of-fit test (see Ait-Sahalia, 1996; Corradi and White, 1999; Dette and von Lieres und Wilkau, 2003). Ait-Sahalia (1996) assumes a time span approaching infinity as the sample size increases and considers the problem of testing a joint parametric specification of drift and variance, while in the other references a fixed time span is considered, where the discrete sampling interval approaches zero, and a parametric hypothesis regarding the volatility function is tested. This modeling might be more appropriate for high-frequency data.

In the present paper we also consider the case of discretely observed data on a fixed time span, say $[0, 1]$, from model (1) with increasing sample size. As pointed out by Corradi and White (1999) this model is appropriate for analyzing the pricing of European, American or Asian options. These authors consider the sum of the squared differences between a nonparametric and a parametric estimate of the variance function at a fixed number of points in the interval $[0, 1]$. Although this approach is attractive because of its simplicity, it has been argued by Dette and von Lieres und Wilkau (2003) that the results of the test may depend on the number and location of the points, where the parametric and nonparametric estimates are compared. Therefore, these authors suggest a new test for the parametric form of the volatility in the diffusion model (1), which does not depend on the state x, i.e. $\sigma(t, X_t) = \sigma(t)$. The test is based on an L^2-distance between the volatility function in the model under the null hypothesis and alternative. This approach yields a consistent procedure against any (fixed) alternative, which can detect local alternatives converging to the null hypothesis at a rate $n^{-1/4}$. In the present paper an alternative test for the parametric form of the volatility function is proposed, which is based on a process of the integrated volatility. Our motivation for considering functionals of stochastic processes as test statistics stems from the fact that tests of this type are more sensitive with respect to Pitman alternatives. Moreover, the new tests are also applicable for testing parametric hypotheses on the volatility, which depend on the state x.

In Section 2 we introduce some basic terminology and describe two kinds of parametric hypotheses for the volatility function. We also define two types of stochastic processes of the integrated volatility, which will be used for the construction of test statistics for these hypotheses. Section 3 contains our main results. We show convergence in probability of the stochastic processes to a random variable, which vanishes if and only if the null hypothesis is satisfied. Moreover, we also establish weak convergence of appropriately scaled processes of the integrated volatility to a centered process under the null hypothesis of a parametric form of the volatility. Consequently, the Kolmogorov–Smirnov and Cramér von Mises functional of these processes are natural test statistics. In general the limiting process is a complicated "function" of the data generating diffusion, but conditioned on the diffusion $(X_t)_{t\in[0,1]}$ it is a Gaussian process. In the problem of testing for homoscedasticity these tests are asymptotically distribution free and the limit distribution is given by a Brownian bridge. In Section 4 we study the finite sample properties of the proposed methodology and compare the new procedure with the currently available tests for the parametric form of the volatility function. For high-frequency data the new tests yield a reliable approximation of the nominal level and substantial improvements with respect to power compared to the currently available procedures. Finally, all proofs and some auxiliary results are presented in an Appendix.

2. Specification of a parametric form of the volatility

Let $(W_t)_{t\geqslant 0}$ denote a standard Brownian motion defined on an appropriate probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0\leqslant t\leqslant 1}, P)$ with corresponding filtration $\mathscr{F}_t^W = \sigma(W_s, 0\leqslant s\leqslant t)$ and assume that the drift and variance functions in the stochastic differential equation (1) are locally Lipschitz continuous, i.e. for every integer $M>0$ there exists a constant K_M such that

$$|b(t,x) - b(t,y)| + |\sigma(t,x) - \sigma(t,y)| \leqslant K_M|x-y| \tag{2}$$