Available online at www.sciencedirect.com

ScienceDirect

Journal of Econometrics 143 (2008) 206–225

JOURNAL OF Econometrics

www.elsevier.com/locate/jeconom

Diagnostic testing for cointegration

P.M. Robinson*

Department of Economics, London School of Economics, Houghton Street, London WC2A 2AE, UK

Available online 2 September 2007

Abstract

We develop a sequence of tests for specifying the cointegrating rank of, possibly fractional, multiple time series. Memory parameters of observables are treated as unknown, as are those of possible cointegrating errors. The individual test statistics have standard null asymptotics and are related to Hausman specification test statistics: when the memory parameter is common to several series, an estimate of this parameter based on the assumption of no cointegration achieves an efficiency improvement over estimates based on individual series, whereas if the series are cointegrated the former estimate is generally inconsistent. However, a computationally simpler but asymptotically equivalent approach, which avoids explicit computation of the ‘‘efficient’’ estimate, is instead pursued here. Two versions of it are initially proposed, followed by one that robustifies to possible inequality between memory parameters of observables. Throughout, a semiparametric approach is pursued, modelling serial dependence only at frequencies near the origin, with the goal of validity under broad circumstances and computational convenience. The main development is in terms of stationary series, but an extension to non-stationary ones is also described. The algorithm for estimating cointegrating rank entails carrying out such tests based on potentially all subsets of two or more of the series, though outcomes of previous tests may render some or all subsequent ones unnecessary. A Monte Carlo study of finite sample performance is included.

JEL classification: C32

Keywords: Fractional cointegration; Diagnostic testing; Specification testing; Cointegrating rank; Semiparametric estimation

1. Introduction

The potential for detecting cointegration in economic and financial time series has expanded with a wider realization that the phenomenon is not restricted to the ‘‘unit root setting’’ of $I(1)$ observable series, where cointegrating errors are $I(0)$ (or to its familiar extensions to $I(2)$ series, or $I(1)$ series with deterministic trends). Fractional processes provide a significant mathematical extension of these. We say that a $p \times 1$ vector z_t of jointly dependent $I(\delta)$ processes, for some positive, real integration order δ that need not be an integer, is cointegrated if there exists a linear combination of elements of z_t that is $I(\gamma)$, for some real $\gamma \in [0, \delta)$. Indeed, z_t can even be stationary, in which case $\delta < \frac{1}{2}$. As in the traditional unit root setting, there could be up to $p - 1$ cointegrating relations, however, we permit these to have real-valued and possibly different integration orders.

*Tel.: +44 20 7955 7516; fax: +44 20 7955 6592.
E-mail address: p.m.robinson@lse.ac.uk

0304-4076/$ - see front matter
doi:10.1016/j.jeconom.2007.08.015

It is useful to estimate cointegrating relations; these can be used to test hypotheses of interest (such as PPP) and to improve the precision of forecasts, for example. However, important initial questions are the existence of cointegration, and the cointegrating rank, that is the number of cointegrating relations. These have been addressed in the unit root setting, e.g. by Johansen (1988, 1991, 1996) and Phillips and Ouliaris (1988, 1990). Here it has been usual to take for granted the $I(1)$ assumption on z_t, albeit with the presumption of pre-testing. Procedures based on known integration orders can be invalidated if they are mis-specified. With sufficiently long series, it may thus seem more satisfactory to adopt an agnostic approach, by estimating integration orders from the data, though estimates of course cannot be taken as synonymous with true values, and it seems desirable to investigate the existence of cointegration in the presence of unknown δ (and γ). There has been some, rather limited, theoretical investigation of this problem (see e.g. Robinson and Yajima, 2002).

The present paper studies a conceptually and computationally simple diagnostic statistic, based on the Hausman specification testing idea, for testing the null hypothesis of no cointegration. A version of it was previously proposed (in bivariate series) by Marinucci and Robinson (2001), but though they applied it empirically its theoretical properties were only heuristically discussed. Here we not only provide a more formal treatment of its asymptotic null distribution, but also discuss its consistency properties, and its robustness to departures from mainstream assumptions; this in particular leads us to propose a robustified version. We propose an algorithm for estimating cointegrating rank in series of dimension 3 or more, which uses one of our tests in a sequential manner on each step. To prevent the discussion becoming too complicated we do not incorporate deterministic trends, indeed for partly expository purposes the main discussion is in terms of stationary observables, though we later describe an extension to non-stationary series.

The following section describes a stationary setting in which cointegration, with some rank, may or may not occur. Section 3 defines two basic test statistics, which assume integration orders of all observables are equal. Their asymptotic null distributions, and the consistency properties of one of them, are presented in Section 4. Section 5 introduces and justifies a robustified statistic which avoids the assumption of equality of all integration orders and includes an asymptotic local power comparison. Section 6 presents the algorithm for estimating cointegrating rank. Section 7 describes an extension to non-stationary series. A Monte Carlo study of finite-sample performance is reported in Section 8. Section 9 offers some brief final comments.

2. Fractional cointegration

We assume initially an observable p-dimensional column vector z_t, $t = 0, \pm 1, \ldots,$ of jointly stationary series that all have the same, unknown, integration order $\delta \in (0, \frac{1}{2})$. By this we mean, denoting by $f_z(\lambda)$, $\lambda \in (-\pi, \pi]$, the spectral density matrix of z_t, that

$$f_z(\lambda) \sim G_0 \lambda^{-2\delta} \quad \text{as } \lambda \to 0+, \tag{2.1}$$

where "$\sim$" means throughout that the ratio of real parts, and of imaginary parts, of corresponding elements on the left- and right-hand side, tends to 1, and the $p \times p$ real matrix G_0 is positive semidefinite with positive diagonal elements.

We say that z_t is cointegrated with rank $r<p$ if there exist r linearly independent $p \times 1$ vectors β_i such that for $i = 1, \ldots, r$, $u_{it} = \beta_i' z_t$ has spectral density $f_{u_i}(\lambda)$ satisfying

$$f_{u_i}(\lambda) \sim \omega_{ii} \lambda^{-2\gamma_i} \quad \text{as } \lambda \to 0+, \tag{2.2}$$

where

$$0 \leqslant \gamma_1, \gamma_2, \ldots, \gamma_r < \delta; \quad \omega_{ii} > 0, \ i = 1, \ldots, r. \tag{2.3}$$

For the convenience of a simple notation for all relevant integration orders, we also introduce

$$\gamma_i = \delta, \quad i = r+1, \ldots, p. \tag{2.4}$$

We can embed the cointegration within a non-singular system of degree p, writing

$$B z_t = u_t, \tag{2.5}$$