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A non-parametric independence test using permutation entropy

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Abstract

In the present paper we construct a new, simple, consistent and powerful test for independence by using symbolic dynamics and permutation entropy as a measure of serial dependence. We also give a standard asymptotic distribution of an affine transformation of the permutation entropy under the null hypothesis of independence. The test statistic and its standard limit distribution are invariant to any monotonic transformation. The test applies to time series with discrete or continuous distributions. Eventhough the test is based on entropy measures, it avoids smoothed non-parametric estimation. An application to several daily financial time series illustrates our approach.

Keywords: Entropy; Independence; I.i.d; Invariance; Nonlinear time series; Symbolic dynamics; Random walk

1. Introduction

Independence is one of the most valuable notions in econometrics, time series analysis and statistics due to the fact that most tests boil down to checking some sort of independence assumption. As a result, an extensive literature on how to test independence has arisen: Correlation tests (see King, 1987 for a survey) are widely used, but they are not consistent against alternatives with zero autocorrelation. Examples of serially dependent processes that exhibit zero autocorrelation include autoregresive conditional heteroskedastic (ARCH), bilinear, nonlinear moving average processes and iterative logistic maps. The non-parametric literature also contains a large number of serial independence test (see Dufour et al., 1982 for a bibliographical survey on permutation, sign and rank tests for independence). These test procedures work well under commonly used dependence structures, like ARMA models, but they also fail to detect subtle nonlinear underlying dependence structures. Needless to say that other non-parametric tests have emerged (Brock et al., 1996; Pinkse, 1998, among others) to cover these difficulties.

Serial independence has been increasingly studied by using entropy measures. These measures avoid restrictive parametric assumptions on the probability distribution generating the data, and they can capture the dependence present in a time series. As an outcome, establishing asymptotic distribution theory for smoothed non-parametric entropy measures of serial dependence has been so far challenging (see Hong and

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White, 2005 and references therein). This line of research is narrowly connected with *Information Theory*: Jaynes (1957) introduced the maximum entropy principle (MEP) which determines the probability distribution of a random variable by maximizing the Shannon entropy, subject to certain moment conditions. This optimization principle is the same as the Kullback principle of minimizing the Kullback–Leibler relative entropy when one of the distributions is uniform. Jaynes' MEP was a turning point in the use of Shannon's entropy as a method of statistical inference.

The use of entropy has played a leading role as a measure of the dependence present in a time series in the last two decades. Joe (1989a,b) considered a smoothed non-parametric entropy measure of multivariate dependence of an independent and identically distributed (i.i.d.) random vector. Granger and Lin (1994) proposed a normalized smoothed non-parametric entropy measure of serial dependence to identify important lags in time series. Robinson (1991) developed a test for serial dependence using a modified entropy measure. Skaug and Tjøstheim (1993b, 1996) also considered a general class of smoothed density-based tests for serial dependence, which includes a test based on an entropy measure modified with a weight function.

As Granger and Lin (1994) pointed out, there is no asymptotic distribution theory available for smoothed non-parametric entropy measures of serial dependence. Consistency and in some cases convergence rates have been established, but asymptotic distributions for these entropy estimators are not available. Robinson (1991) first provided an asymptotic distribution theory for a smoothed non-parametric modified entropy measure of serial dependence, using a sample-splitting device. Granger et al. (2004) introduced a transformed metric entropy of dependence. Recently, the relevant investigation of Hong and White (2005) have provided, under certain assumptions, an asymptotic theory for a class of kernel-based smoothed non-parametric entropy estimators of serial dependence. They also show that their theory yields the limit distribution of the Granger and Lin's normalized entropy measure, which was previously unknown in the literature. Moreover, they develop a test that is asymptotically locally more powerful than Robinson's test. Nevertheless, most of the methods used to test for independence via an entropy measure of serial dependence strictly require a continuous distribution function of the unknown underlying data generating process and also need to estimate the density function with stochastic kernels. As a result, free-choice parameters are introduced. Another difficulty acknowledged by Hong and White (2005) is that the finite sample level of their own test (and in general of others entropy-based tests) differs from the asymptotic one; furthermore, asymptotic theory may not work well even for relatively large samples. This leads to implement, for each sample size, non-naive bootstrap procedures in order to correctly compute the test. Moreover, Hong and White need the time series $\{X_t\}$ to have a compact support in the interval [0, 1], although this is not necessarily a restriction whenever the test is invariant under monotonic transformations of the series. Obtaining a compact support can always be ensured by a continuous strictly monotonic transformation such as the logistic function.[1]

On the other hand, there are a number of other non-paremetric tests for independence that avoid smoothed non-paremetric estimation (Skaug and Tjøstheim, 1993a; Delgado, 1996; Hong, 1998, 2000; see also Tjøstheim, 1996 for an excellent complete survey). These procedures are based on the empirical distribution function or on the characteristic function. Importantly, some of these statistics are invariant to order preserving transformation; the distribution generating the data can be continuous or discrete; under certain conditions, the tests are distribution free. Unfortunately, some of these statistics have non-standard limiting distribution. In these procedures, as in the case of tests based on smoothing estimation techniques, the test statistic is a distance between the joint density (or estimated joint distribution) and the marginal densities (or estimated marginal distributions).

In the present paper we take a different way, and we propose a new test for independence also based on *Information Theory*, but avoiding the potential disadvantage of depending on the choice of a smoothing number. More precisely, the absence of dependencies in the unknown underlying data generating process is studied via symbolic dynamics. Symbolic dynamics studies dynamical systems on the basis of the symbol sequences obtained for a suitable partition of the state space. The basic idea behind symbolic dynamics is to divide the phase space into a finite number of regions and label each region by an alphabetical letter. In this regard, symbolic dynamics is a coarse-grained description of dynamics. Some recent tests of independence are also a coarse-grained description of the underlying dynamic from which the data was generated. Even though

[1] $X_t^* = 1/1 + exp^{-X_t}$, where X_t is the observed data point and X_t^* its corresponding logistic transformation.