Computers and Structures 171 (2016) 31–45

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

State-space based time integration method for structural systems involving multiple nonviscous damping models

Zhe Ding, Li Li, Yujin Hu *, Xiaobai Li, Weiming Deng

State Key Laboratory of Digital Manufacturing Equipment and Technology, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 11 January 2016
Accepted 18 April 2016
Available online 17 May 2016

Keywords:
Nonviscous damping
Multiple damping models
Time integration
State-space method
Dynamic response

ABSTRACT

Engineering structures are often assembled by subsections with various levels of energy dissipation, which lead to great difficulties in analyzing. In this paper, a direct time integration method is proposed to solve the dynamic responses of structural systems involving multiple nonviscous damping models. Based on a general non-viscously damped model, an extended state-space scheme for the damped system is derived. Then, an implicit time integration method using linear approximations is developed built on the extended state-space formalism. The stability and efficiency of the method are discussed by theoretical and numerical analyses. The method is illustrated by two examples.

1. Introduction

Damping is a significant characteristic parameter describing the energy-dissipation and longtime regarded as a key factor in the dynamic behavior of vibrating systems. In structure dynamics, the viscous damping model has been widely applied in the past centuries for its conceptual simplicity. The damping force in this model is often assumed to be proportional to velocity. However, applications of sophisticated engineering systems [1–6] and composite materials [7,8] manifest that the classical viscous damping model does not properly represent the complicated damping character in these structures. Therefore, nonviscous damping models [9–19] are proposed in recent years with an aim to describe damping energy-dissipation in a more general manner compared to the limited scope offered by the viscous damping model. The nonviscous damping models assume that the damping forces depend on the past history of motion via convolution integrals over some decaying kernel functions. As a result, classical dynamic analysis methods for the viscous damping model cannot be applied directly to the nonviscously damped systems and new methods are needed to be developed.

In recent years, researchers have studied the dynamic responses of the nonviscously damped systems. In general, any modifications of kernel function may lead to some mathematical difficulties and changes for solving the corresponding dynamic equations. Some authors developed methods in an attempt to efficiently calculate the eigenproblems of the nonviscously damped systems under different damping models. The methods can be classified as exact state-space approaches [20–24], approximated methods [25–32] and model reduction methods [4,33–38]. Unfortunately, the aforementioned methods are still time-consuming due to the eigenvalue problems involved. Besides, some frequency-domain methods [39,40] are also proposed to analyzing the exponentially damped linear systems, but the efficiency and accuracy of the frequency-domain methods are restricted by the Laplace and inverse Laplace transform between frequency- and time-domain.

Direct time-integration methods, including implicit methods [41–46] and explicit methods [47–49], have been successfully applied to compute the dynamic responses of the viscous damping systems subjected to complicated dynamic loadings. Therefore, some researchers developed the direct time-integration methods to capture the dynamic responses for viscoelastic damping systems. Muravyov [50–52] proposed free and forced vibration responses methods of nonviscously damped systems in time-domain. Later, Adhikari and Wagner [53] presented a direct time-domain integration method for exponentially damped linear systems based on the extended state-space approach proposed by Wagner and Adhikari [24]. Then, Cortés and Elejabarrieta [26] developed a direct integration formulation by transforming the equation of motion with convolution integrals into a differential equation with time derivative orders higher than two via Laplace transform. The method did not employ any internal variables which normally enlarge the size of the problem, but it can be only

* Corresponding author.
E-mail address: yjhu@mail.hust.edu.cn (Y. Hu).

http://dx.doi.org/10.1016/j.compstruc.2016.04.002
0045-7949/

used in the case when exponential damping kernel function no higher than two. However, the previously mentioned time-domain methods are all restricted to exponential damping kernel. Liu [54] and Puthanpurayil et al. [55] extended implicit methods to obtain the dynamic responses of nonviscous damping systems. The methods are based on Newmark integration method and generic to all kernel functions. Lately, Liu [56] developed explicit method of dynamic response to nonviscously damped systems. The method is proven more efficient than the implicit methods [54,55], but the stable conditions are not clear and need to be further investigated.

Nowadays, with the rapid development of modern science and technology, engineering structures become increasingly complex and large. Two or more subcomponents with significantly different levels of energy dissipation are encountered frequently in dynamical designs. Thus, these damping systems often involve multiple damping models. Unfortunately, existing computational methods for damped systems are only dealing with the dynamic analysis of the system with only one damping models and may have some limitations to handle the problem if two or more damping models are considered. Recently, Li and Hu [57] proposed a unified way to express damping models by using a fraction formula of rational polynomials. Then, a state-space approach for the analysis of linear systems with multiple damping models have also developed. The dynamic responses of the system were calculated by mode-superposition method using eigensolutions obtained by the state-space method. Although the method gives exact results when considering all elastic and nonviscous modes, it requires tremendous computational time, especially in the case involving multiple damping models.

Due to the problems mentioned above, a time-domain approach for multiple nonviscous damping models is proposed. By introducing a general nonviscously damped model, an extended state-space formulation is derived for the generalized damped systems. Based on this, an implicit time integration method is developed using linear approximations of displacements, velocities and internal variables. The method is proven to be unconditionally stable. The advantage of the proposed method over the traditional mode superposition method is that the final recurrence formulas to be solved contain nothing but a linear combination of the system matrices. It will be demonstrated by two numerical examples that the proposed method is more efficient and accurate compared to other methods.

The paper is organized as follows: in Section 2, we will briefly review some preliminary concepts and definitions of multiple damping models. In Section 3, We will introduce the mode superposition method for viscoelastically damped systems and extend the implicit method to be applicable to multiple damping models. Then in Section 4, A direct time-domain approach for multiple damping models is proposed to calculate the dynamic response of the system. Besides, the stability condition is also discussed and the summary of the method will be displayed in Section 4 to make it convenient to be coded up. The results and discussions of two numerical examples are presented in Section 5 in order to assess the performance of the proposed method. The paper ends with some conclusions in Section 6.

2. Theoretical background

The equations of motion of an N-degree-of-freedom (DOF) linear system with nonviscous damping, which depend on the past history of motion via convolution integrals over kernel functions, can be expressed by [24,54,56,58]

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \int_0^t g(t-\tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

together with the initial conditions

$$\mathbf{x}(t=0) = \mathbf{x}_0 \in \mathbb{R}^N, \quad \dot{\mathbf{x}}(t=0) = \dot{\mathbf{x}}_0 \in \mathbb{R}^N \tag{2}$$

Here $\mathbf{M} \in \mathbb{R}^{N\times N}$ is the mass matrix, $\mathbf{K} \in \mathbb{R}^{N\times N}$ is the stiffness matrix, $\mathbf{x}(t) \in \mathbb{R}^N$ is the displacement vector, $\mathbf{f}(t) \in \mathbb{R}^N$ is the forcing vector and $g(t-\tau)$ is the matrix of damping kernel function. In special cases, when $g(t-\tau) = \mathbf{C}\delta(t-\tau)$, where $\delta(t)$ is the Dirac delta function and $\mathbf{C}$ is a constant matrix, Eq. (1) reduces to the case of viscous damping system. Therefore, the viscoelastic damping model is considered as a further generalization of the familiar viscous damping.

In theory, any mathematic model is a possible candidate for a nonviscous model as long as it can make the energy dissipation functional nonnegative. Some well-known damping models are derived by authors to accurately describe the dissipative mechanisms of viscoelastic materials. Exponential damping model is widely used to model the viscoelastic damping system [23,24]

$$g(t) = \sum_{k=1}^{m} c\mu_k \exp(-\mu_k t) \tag{3}$$

or, in the Laplace domain

$$G(s) = \sum_{k=1}^{m} \frac{c\mu_k}{s+\mu_k} \tag{4}$$

where constants $c \in \mathbb{R}^+, \mu_k \in \mathbb{R}^+ (k = 0,1,2,\ldots)$ are known as the relaxation parameters, and m denotes the number of relaxation parameters used to describe the viscoelastic damping behavior.

The Biot model [9,10] is denoted by

$$g(t) = a_0\delta(t) + \sum_{k=1}^{m} a_k \exp(-b_k t) \quad \text{or} \quad G(s) = a_0 + \sum_{k=1}^{m} \frac{a_k}{s+b_k} \tag{5}$$

where constants $a_k, b_k \in \mathbb{R}^+ (k = 0,1,2,\ldots)$ are known as the relaxation parameters.

The Golla–Hughes–McTavish (GHM) model [11,12] is expressed as

$$g(t) = G_0 \sum_{k=1}^{m} \alpha_k \frac{\hat{b}_{2k}e^{-\hat{b}_{1k}t} - \hat{b}_{1k}e^{-\hat{b}_{2k}t}}{\hat{b}_{2k} - \hat{b}_{1k}} \quad \text{or}$$
$$G(s) = G_0 \sum_{k=1}^{m} \alpha_k \frac{s^2 + 2\hat{\xi}_k\hat{\omega}_k s}{s^2 + 2\hat{\xi}_k\hat{\omega}_k s + \hat{\omega}_k^2} \tag{6}$$

where constants $G_0, \hat{b}_{1k}, \hat{b}_{2k}, \hat{\omega}_k, \hat{\zeta}_k \in \mathbb{R}^+$ are also known as the relaxation parameters and the relationships of the parameters are

$$\hat{b}_{1k} = \hat{\omega}_k\hat{\xi}_k - \hat{\omega}_k\sqrt{\hat{\xi}_k^2 - 1}, \quad \hat{b}_{2k} = \hat{\omega}_k\hat{\xi}_k + \hat{\omega}_k\sqrt{\hat{\xi}_k^2 - 1} \tag{7}$$

The Anelastic Displacement Field (ADF) model [13,14] express the damping kernel function as

$$g(t) = \sum_{k=1}^{m} \Delta_k \exp(-\Omega_k t) \quad \text{or} \quad G(s) = \sum_{k=1}^{m} \frac{\Delta_k}{s+\Delta_k} \tag{8}$$

where constants $\Delta_k, \Omega_k \in \mathbb{R}^+$ are relaxation parameters.

Some authors also consider other damping models, such as the generalized Maxwell model [15], the fractional derivative model [18] and the Gaussian damping model [19] (see e.g., Ref. [59] for further reading). However, for Eq. (1), no classical numerical methods can be used directly for its solution of dynamic responses. Besides, any modifications of the kernel functions may lead to some mathematical difficulties and changes for solving the corresponding dynamic equations. As a result, some researchers [1,30,39,53,60,61] devoted to calculate the dynamic response under different viscoelastic damping systems. Unfortunately, the majority of the proposed methods are only restricted to systems with only one specific damping model, which have some