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Nonlinearity, nonstationarity, and spurious forecasts

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Abstract

Implications of nonlinearity, nonstationarity, and misspecification are considered from a forecasting perspective. Our model allows for small departures from the martingale difference sequence hypothesis by including a nonlinear component, formulated as a general, integrable transformation of the $I(1)$ predictor. We assume that the true generating mechanism is unknown to the econometrician and he is therefore forced to use some approximating functions. It is shown that in this framework the linear regression techniques lead to spurious forecasts. Improvements of the forecast accuracy are possible with properly chosen nonlinear transformations of the predictor. The paper derives the limiting distribution of the forecasts' mean squared error (MSE). In the case of square integrable approximants, it depends on the L_2-distance between the nonlinear component and approximating function. Optimal forecasts are available for a given class of approximants.

JEL classification: C22; C53; G14

Keywords: Forecasting; Integrated time series; Misspecified models; Nonlinear transformations; Stock returns

1. Introduction

Nonlinear models are extensively used in econometrics (for a description and analysis of various nonlinear models see, for example, Granger and Teräsvirta, 1993). The theoretical foundation for estimation of nonlinear, nonstationary models has been developed fairly recently. Park and Phillips (1999) derived asymptotic results for the sums of nonlinear transformations of integrated time series. They considered three classes of nonlinear functions: integrable, homogeneous, and exponential. They show, for example, that partial sums of integrable functions that have a nonzero Lebesgue measure converge in distribution to local times of the Brownian motion. Their results have been applied to various nonlinear econometric models. Chang et al. (2001) considered nonlinear regression with separably additive regression functions. Chang and Park (2003) considered nonstationary index models, which extend switching regressions to the stochastic trends framework. Hu and Phillips (2004) studied nonstationary discrete choice models. Kasparis (2004) considered effects of functional form misspecification on estimation, when the true and estimated models involve nonlinearity and nonstationarity. He focused on convergence of estimators to some pseudo-true

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values, and detection of functional form misspecification. Hong and Phillips (2005) develop a linearity test of cointegrating relations.

An attractive feature of nonlinear models is flexibility that allows one to model relationships between nonstationary and seemingly stationary variables. A linear regression requires the dependent variable to have the same order of integration as the right-hand side of the regression equation. However, it is known that nonlinear transformations can change the memory properties of a process. Thus, contrary to linear regressions, properly chosen nonlinear functions can link in a single equation variables that appear to have different orders of integrations. Nonlinear functions that can be used to model relationships between seemingly stationary and persistent variables include Lebesgue integrable functions and asymptotically homogeneous functions of degree zero (the distribution-like functions). For example, Chang and Park (2003) modelled nonstationary switching behavior using a distribution type function with an $I(1)$ variable as the argument.

There are many situations in economics that may require one to relate variables of different orders of integration. A typical example is the predictive regressions literature in empirical finance, which studies stock returns predictability. In a predictive regression, stock returns are regressed on lagged values of various financial and economic variables such as the dividend-price ratio, earnings-price ratio, or interest rates. While most researchers agree that stock market returns are $I(0)$, predictors such as dividend-price ratio appear to have a stochastic trend component.

In predictive regressions, predictability is usually concluded on the basis of t-tests for slope coefficients. Often, it is implicitly assumed that non-zero in-sample correlations found between regressors and stock returns can be used for construction of out-of-sample forecasts. Many papers report statistically significant slope estimates (see, for example, Fama, 1991; Cochrane, 1997 for surveys of the literature). Despite the collected empirical evidence on in-sample relations between stock returns and predictors, the out-of-sample predictability is still a controversial issue. Goyal and Welch (2003, 2004) report that performance of out-of-sample forecasts based on linear regression methods can be rather poor, while Campbell and Thompson (2004) argue that there exists small but economically meaningful out-of-sample predictive power, once restrictions on the coefficients and forecasts are imposed.

The results in this paper imply that significant regression slopes do not necessarily indicate usefulness of the linear regression as a forecasting equation. Our model allows for small departures from the martingale difference sequence (MDS) hypothesis by including an additive nonlinear component, formulated as a general, integrable transformation of the predictor, which is assumed to be $I(1)$. In this model, the signal coming from the nonlinear component is very weak relative to the noise, as implied by the properties of integrable functions and $I(1)$ variables. An integrable function approaches zero at a fast rate as the absolute value of its argument increases. At the same time, a unit root process usually takes on very large negative or positive values. As a result, the signal coming from the predictor (the nonlinear component) is relatively strong only during rare events, when the unit root process visits the neighborhood of zero. Such a generating mechanism provides for predictability only in the extremely short run, which in the stock market example corresponds to a situation where some relevant information may escape the attention of market participants only for very short periods of time. The process modelled in this paper is opposite to that of Kilian and Taylor (2003); they describe nonlinear mean-reversion, however, combined with long-run predictability (they discuss forecasting of exchange rates).

It is natural to assume that the true data generating process (DGP) involving nonlinear dependency is unknown to the econometrician, and he is therefore forced to use some approximating functions. Furthermore, the class of approximants used by the econometrician does not necessarily include the true function. This paper illustrates by the means of analytical asymptotic results that such a combination of nonstationarity, nonlinearity, and misspecification leads to the results often seen in the predictive regressions literature. Consider for example a linear regression, which is the most popular approximating function. We show that, in this case, commonly used diagnostic tools tend to indicate predictive power despite the fact that estimated regression slopes converge to zero in probability. Moreover, we show that the out-of-sample forecasts constructed from a predictive regression asymptotically have the same mean squared error (MSE) as that of constant forecasts equal to the simple historic average of the dependent variable. Hence, spurious forecasts occur: diagnostic tools may indicate usefulness of the predictive model, while, in