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Testing the Markov property with high frequency data

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Abstract

This paper develops a framework to nonparametrically test whether discrete-valued irregularly spaced financial transactions data follow a Markov process. For that purpose, we consider a specific optional sampling in which a continuous-time Markov process is observed only when it crosses some discrete level. This framework is convenient for it accommodates the irregular spacing that characterizes transactions data. Under such an observation rule, the current price duration is independent of a previous price duration given the previous price realization. A simple nonparametric test then follows by examining whether this conditional independence property holds. Monte Carlo simulations suggest that the asymptotic test has huge size distortions, though a bootstrap-based variant entails reasonable size and power properties in finite samples. As for an empirical illustration, we investigate whether bid–ask spreads follow Markov processes using transactions data from the New York Stock Exchange. The motivation lies on the fact that asymmetric information models of market microstructures predict that the Markov property does not hold for the bid–ask spread. We robustly reject the Markov assumption for two out of the five stocks under scrutiny. Finally, it is reassuring that our results are consistent with two alternative measures of asymmetric information.

JEL classification: C14; C52; G10; G19

Keywords: Bid–ask spread; Conditional independence; Duration; Nonparametric testing; U-statistic

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1. Introduction

Despite the innumerable studies in financial economics rooted in the Markov property, there are only two tests available in the literature to check such an assumption: Aït-Sahalia (2000) and Fernandes and Flôres (2004). To build a nonparametric testing procedure, the first test uses the fact that the Chapman–Kolmogorov equation must hold in order for a process to be compatible with the Markov assumption (see also Aït-Sahalia, 2002). Although the Chapman–Kolmogorov representation involves a quite complicated nonlinear functional relationship among transition probabilities of the process, it brings about several advantages. First, estimating transition distributions is straightforward and does not require any prior parameterization of conditional moments. Second, a test based on the whole transition density is obviously preferable to tests based on specific conditional moments. Third, the Chapman–Kolmogorov representation is well defined, even within a multivariate context.

Fernandes and Flôres (2004) develop alternative ways of testing whether discretely recorded observations are consistent with an underlying Markov process. Instead of using the highly nonlinear functional characterization provided by the Chapman–Kolmogorov equation, they rely on a simple characterization out of a set of necessary conditions for Markov models. As in Aït-Sahalia (2000), the testing strategy boils down to measuring the closeness of density functionals that are nonparametrically estimated by kernel-based methods.

Both testing procedures assume, however, that the data are evenly spaced in time. Financial transactions data do not satisfy such an assumption and hence these tests are not appropriate. To design a test for the Markov property that is suitable to high frequency data, we build on the theory of Markov processes with stochastic time changes. We consider a particular optional sampling for the underlying continuous-time Markov process $\{\tilde{X}_t;\ t>0\}$ that yields discrete-time realizations $\{\tilde{X}_{t_1},\ldots,\tilde{X}_{t_n}\}$ as the cumulative change in $\tilde{X}_t$ adds to a discrete level c. Accordingly, we accommodate the irregular spacing that characterizes transactions data. Further, such an optional sampling scheme implies that consecutive spells between the observed changes in $\tilde{X}_t$ are conditionally independent given the discrete-time realization of $\tilde{X}_t$. We then develop a simple nonparametric test for the Markov property by testing whether the conditional independence property holds.

There is a large literature on how to test either unconditional independence (e.g., Hoeffding, 1948; Rosenblatt, 1975; Pinkse, 1999) or serial independence (e.g., Robinson, 1991; Skaug and Tjøstheim, 1993; Pinkse, 1998). However, there are only a few papers discussing tests of conditional independence: Linton and Gozalo (1999) and, more recently, Su and White (2002, 2003a,b). Linton and Gozalo (1999) test for conditional independence between iid random variables by looking at the restrictions on the cumulative distribution function under a quadratic measure of distance. Su and White (2002, 2003a,b) extend their approach by considering weakly dependent stochastic processes as well as different metrics. In particular, Su and White (2002) verify whether the density restriction implied by conditional independence holds using the Hellinger distance, whereas Su and White (2003a,b) check restrictions on the characteristic function and on the empirical likelihoods, respectively. Our setting combines the setups of Linton and Gozalo (1999) and Su and White (2002). As in Su and White (2002), we derive tests under mixing conditions so as to deal with the time-series dependence of the data. However, we