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Multiple-point constraint applications for the finite element analysis of shear deformable composite beams – Variational multiscale approach to enforce full composite action

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ABSTRACT

Composite beams that consist of two or more shear deformable layers find widespread applications in a variety of engineering structures. In the computational modelling of composite beams, the layers can be stacked together and connected conveniently at the nodes by using multiple-point constraints. However, this type of modelling does not inherit the kinematic behaviour of the continuous case and thus full-interaction between the layers cannot be always imposed by applying multiple-point constraints at the nodes. The work herein shows that in multiple-point constraint applications full composite action between the shear deformable layers can be recovered by using the variational multiscale approach. The originality of this study is in the interpretation of the multiple-point constraint application as the solution in a superfluously extended space because of the weakening in the kinematic constraints. It is shown that full composite action between the beam layers can be recovered by excluding the identified fine-scale effect from the solution of the multiple point constraint application. Selected examples illustrate the effects of loading and relative layer stiffness on the numerical error as well as modelling options for fully composite and delaminated beams.

1. Introduction

Composite beams that consist of two or more shear deformable layers stacked together find widespread applications in a variety of engineering structures. Numerous laminated beam theories have been proposed hitherto to describe the kinematics and stress states of composite laminates. The Equivalent Single-Layer models replace the heterogeneous laminate with a single layer whose stiffness is a weighted average of the layer stiffnesses thorough the thickness. Such models are the simplest of all laminate theories and are generally suitable to predict the global response [1]. Alternatively, layer-wise theories assume separate displacement fields within each layer, which provides a kinematically more flexible representation. As a result, such models are capable of predicting the strain and stress fields in each layer more accurately. Detailed discussions and reviews on several layer-wise composite laminated beam theories can be found in [2–4]. These alternative layer-wise theories vary based on the kinematics assumptions adopted in each layer and the way that the displacement continuity is enforced between the layers. The elementary theory of bending of beam based on Euler–Bernoulli kinematics is adoptable only for thin laminates and materials with high transverse shear modulus. Therefore, a more improved beam theory of Timoshenko (i.e., First-Order Shear Deformation Theory) is widely adopted for composite laminates because of its shear deformability and usability for thick laminates. In the present study, the analysis is based on the assumption that each layer acts as a shear deformable Timoshenko beam. Also, the displacement continuity is enforced between the layers such that a single transverse deflection describes the transverse deflection across the thickness (i.e., transverse incompressibility and no vertical separation) and a single rotation angle describes the rotation across the thickness (i.e., the composite cross-section remains planar after the deformation). In this study, as a practical modelling approach, Multiple-Point Constraints (MPCs) are used at the nodes to enforce continuity between the layers. This approach to form composite beams allows easy modelling, especially when separation between the layers and delamination needs to be considered at some parts of the beam. However, when the intention is to provide full interaction between the layers, the composite beam model that is generated by using MPCs between the nodes of separate layers does not necessarily inherit the kinematic behaviour of the continuous case. Gupta and Ma [5] pointed out this issue in composite beams composed

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of Euler–Bernoulli beam layers and noted that the source of error in MPC applications of this type can be related to the incompatibility in the displacement field. Numerical issues in master–slave type constraint applications of this type were also investigated in [6,7]. Another type of numerical error occurs in two node beam formulations when the reference axis of the beam-type finite element does not match with the neutral axis of the cross-section which is often the case in multi-layer composite beam finite element formulations. Casaux-Ginestet and Ibrahimbegovic [8] identified this issue also as an incompatibility issue between the axial and bending deflections and corrected this incompatibility by enriching the axial displacement field by using bubble functions. In Erkmen et al. [6], variational multiscale approach was used as a paradigm to identify both types of numerical issues, i.e. issues in MPC applications as well as in eccentric axis selections in beam formulations.

The variational multiscale paradigm was introduced by Hughes and his colleagues to present a theoretical frame-work for the applications of stabilized techniques such as enrichment with bubble functions to avoid spurious numerical constraints in the finite element method (e.g., [9,10]) as well as for the applications of regularization techniques to avoid mesh dependency in localized failure problems such as strain softening. Some applications of the multiscale paradigm in complex localized failure problems can be found in [11–15]. In this study, the variational multiscale approach is used as a paradigm to identify the numerical error in MPC applications to form a shear deformable composite beam. It is shown that the interpolation space in the MPC application can be treated as a superfluously extended space. By using the variational multiscale approach, firstly, the incompatibility in the interpolations of the displacements, that occurs when enforcing continuity between the layers, is avoided in the finite element formulation and thus, full composite action between the layers is recovered by excluding the fine-scale effect from the solution of the MPC application. Interestingly, the targeted solution is initially the coarse-scale solution in the problem considered herein. After application of the MPCs, an equivalent two node composite beam element is obtained. In such cases when the reference axis of the element (after MPC application) does not match with the neutral axis of the cross-section, the element can also be modified to obtain accurate results as pointed out in [8] and this is done herein again by using the variational multiscale approach second time in a standard manner, in which the targeted solution is the fine-scale solution in this case. The improvements in the accuracy and convergence characteristics are illustrated with numerical examples. It is also shown that the effects of the incompatibility can be represented by using extra fictitious elements and springs, which offers a direct correction technique that is especially useful when the access to the numerical procedure is limited.

The paper is organised as follows. The kinematics and the weak form of the equilibrium equations for the composite beam layers are introduced in Section 2. In Section 3, finite element formulations are developed for composite beam analysis by using MPCs and alternatively by enforcing full composite action between the layers as *a priori* condition. In Section 4, it is shown that by using the variational multiscale approach the finite element formulation based on full composite action can be recovered from the formulation based on MPC application. In Section 5, a finite element formulation that provides exact values at the nodes is obtained also by using the variational multiscale approach. Numerical examples are presented in Section 6 and conclusions are drawn in Section 7.

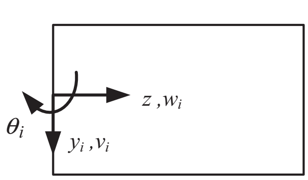

Fig. 1. Positive in-plane displacement and coordinate directions of layer *i*.

2. Composite beam kinematics and finite element solution

2.1. Displacements and strains

The composite beam is made up of n layers in which an arbitrary layer is referred to as layer i, i.e. $i = 1, 2, \ldots, n$. According to shear deformable Timoshenko beam kinematics, the deformations of layer i can be expressed in terms of the axial displacement w_i, transverse displacement v_i and the rotation of the cross-section θ_i. Positive directions for w_i, v_i and θ_i are shown in Fig. 1. The axial strain ε_i in layer i can be determined in terms of the axial displacement gradient Dw_i, and the curvature due to bending $D\theta_i$ as

$$\varepsilon_i = Dw_i - y_i D\theta_i \tag{1}$$

in which y_i is the coordinate of a point on the cross-section with respect to the centroid of layer i and $D() = d()/dz$, where z refers to the axial coordinate of the beam. The shear strain γ_i in layer i can be written as

$$\gamma_i = Dv_i - \theta_i \tag{2}$$

2.2. Weak form of the equilibrium equations

A displacement-based finite element formulation can be developed by employing the principle of virtual work, i.e.

$$\delta\Pi = \sum_{i=1}^{n} \int_L \int_{A_i} \delta\varepsilon_i \sigma_i \, dA\, dz + \sum_{i=1}^{n} \int_L \int_{A_i} \delta\gamma_i \tau_i \, dA\, dz - \delta\Pi_{ext} = 0, \tag{3}$$

where the first integral is the virtual work done due to the bending and axial deformations of the layers, second integral is the virtual work done due to the shear deformations of the layers and $\delta\Pi_{ext}$ is the virtual work done by the external forces. In Eq. (3), σ_i and τ_i are the normal and shear stress configurations in layer i, respectively, which in general can be related with the strain configurations ε_i and γ_i through elasticity and shear moduli, i.e. $\sigma_i = E_i\ \varepsilon_i$ and $\tau_i = G_i\gamma_i$. In Eq. (3), A_i is the cross-sectional area, and L is the span of the beam. Routinely, by substituting Eqs. (1) and (2) into Eq. (3), the weak form of equilibrium equations can be written as

$$\int_L \delta\partial\mathbf{u}^{\mathbf{T}}(z)\mathbf{D}\partial\mathbf{u}(z)\, dz - \int_L \delta\mathbf{u}^{\mathbf{T}}(z)\mathbf{q}(z)\, dz = 0, \tag{4}$$

where **D** is the matrix of the cross-sectional properties, i.e.

$$\mathbf{D} = \begin{bmatrix} E_1A_1 & & & & & & & & & & \\ & E_1I_1 & & & & & & & & & \\ & & \kappa G_1A_1 & & & & & & & & \\ & & & \cdots & & & & & & & \\ & & & & E_iA_i & & & & & & \\ & & & & & E_iI_i & & & & & \\ & & & & & & \kappa G_iA_i & & & & \\ & & & & & & & \cdots & & & \\ & & & & & & & & E_nA_n & & \\ & & & & & & & & & E_nI_n & \\ & & & & & & & & & & \kappa G_nA_n \end{bmatrix}, \tag{5}$$

in which non-diagonal terms are zeros. In Eq. (5), $I_i = \int_{A_i} y_i^2\, dA$ is the moment of inertia of the cross-section of layer i with respect to the bending axis passing through the centroid of layer i and κ is