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HAC estimation in a spatial framework

Harry H. Kelejian*, Ingmar R. Prucha

Department of Economics, University of Maryland, College Park, MD 20742, USA

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Abstract

We suggest a non-parametric heteroscedasticity and autocorrelation consistent (HAC) estimator of the variance–covariance (VC) matrix for a vector of sample moments within a spatial context. We demonstrate consistency under a set of assumptions that should be satisfied by a wide class of spatial models. We allow for more than one measure of distance, each of which may be measured with error. Monte Carlo results suggest that our estimator is reasonable in finite samples. We then consider a spatial model containing various complexities and demonstrate that our HAC estimator can be applied in the context of that model.

JEL classification: C12; C14; C21

Keywords: Heteroscedasticity and autocorrelation consistent (HAC) estimator; Instrumental variable estimator; Spatial models

1. Introduction[1]

Spatial models are an important tool in economics, regional science and geography in analyzing a wide range of empirical issues.[2] Typically, these models focus on spatial interactions, which could be due to competition between cross sectional units, copy-cat policies, net work issues, spill-overs, externalities, regional issues, etc. Applications in the

*Corresponding author. Tel.: +1 301 405 3492; fax: +1 301 405 3542.
E-mail address: kelejian@econ.umd.edu (H.H. Kelejian).
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[2]Classic references on spatial models are Cliff and Ord (1973, 1981), Anselin (1988), and Cressie (1993).

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recent literature include, for example, the determinants of various forms of productivity, various categories of local public expenditures, vote seeking and tax setting behavior, population and employment growth, contagion problems, and the determinants of welfare expenditures.[3] To facilitate the empirical analysis of spatial issues the formal development of estimation methods for spatial models has received increasing attention in recent years.[4]

The purpose of this paper is two-fold: First we suggest, within a spatial context, a non-parametric heteroscedasticity and autocorrelation consistent (HAC) estimator of a variance–covariance (VC) matrix for a vector of sample moments of the form $n^{-1/2}H'u$, where H is a non-stochastic matrix, u is a vector of disturbances, and n is the sample size—i.e., a spatial HAC, (SHAC). The need to estimate the VC matrix of such a vector of sample moments arises frequently within the context of instrumental variable (IV) estimation. We demonstrate the consistency of our SHAC estimator under a set of relatively simple assumptions that cover, in particular, the important and widely used class of Cliff–Ord type models.

HAC estimators have been the focus of extensive research in the time series literature. A classic reference in that literature is Grenander and Rosenblatt (1957). Contributions to this research in the econometrics literature include, among others, Newey and West (1987), Gallant and White (1988), Andrews (1991), Andrews and Monahan (1992), Pötscher and Prucha (1997) and de Jong and Davidson (2000).

In the statistics literature Priestley (1964) made early contributions towards an extension of HAC estimation for spatial processes within the context of estimating spectral densities of stationary random fields (with the index an element of $\mathbf{Z}^2$). The theoretical econometrics literature relating to HAC estimators for spatially dependent data is relatively sparse. To the best of our knowledge, the first contributions to the theoretical econometrics literature are Conley (1996, 1999). However, the approach we take in this paper differs from that of Conley in important ways. Conley assumes that the underlying data generating process is represented by continuous-index random field (with the index an element of a metric space), and explicitly models sampling from this process. He assumes that the data generating process is spatially stationary and spatially alpha mixing. Our setup is different and aims, among other things, to accommodate spatial processes that are generated by Cliff–Ord type models. Those models do not explicitly index observations in terms of elements of a metric space (although they can accommodate such interpretations) and generate the observations as the solution of a simultaneous equation system. Spatial dependences are modeled in terms of a so-called spatial weights matrix. Even if the underlying innovations are i.i.d., this will in general result in a spatial process that is non-stationary simply if the respective units have different numbers of neighbors, as is frequently the case in applications.[5] Our dependence assumptions are stated in terms of

[3]Some applications along these lines are, e.g., Audretsch and Feldmann (1996), Bell and Bockstael (2000), Bernat (1996), Besley and Case (1995), Bollinger and Ihlanfeldt (1997), Buettner (1999), Case (1991), Case et al. (1993), Dowd and LeSage (1997), Holtz-Eakin (1994), Kelejian and Robinson (2000, 1997, 1993), Pinkse et al. (2002), Pulvino (1998), Rey and Boarnet (2004), Shroder (1995), and Vigil (1998).

[4]Recent theoretical contributions include Baltagi and Li (2004, 2001a,b), Baltagi et al. (2003), Conley (1999), Kelejian and Prucha (2004, 2002, 2001, 1999, 1998, 1997), Kelejian et al. (2004), Lee (2004, 2003, 2002, 2001a,b), LeSage (2000, 1997), Pace and Barry (1997), Pinkse and Slade (1998), Pinkse et al. (2002), and Rey and Boarnet (2004).

[5]This is consistent with the view of, e.g., Fuentes (2002a,b) who states that spatial processes are often "non-stationary, in the sense that the spatial structure depends on location". Of course, there are also many situations where stationarity is appropriate and our setup allows for a wide set of stationary processes.