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Modeling ultrasonic waves in elastic waveguides of arbitrary cross-section embedded in infinite solid medium

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ABSTRACT

An approach is presented to model elastic waveguides of arbitrary cross-section coupled to infinite solid media. The formulation is based on the scaled boundary-finite element method. The surrounding medium is approximately accounted for by a dashpot boundary condition derived from the acoustic impedances of the infinite medium. It is discussed under which circumstances this approximation leads to sufficiently accurate results. Computational costs are very low, since the surrounding medium does not require discretization and the number of degrees of freedom on the cross-section is significantly reduced by utilizing higher-order spectral elements.

1. Introduction

This paper presents an extension of a previously developed approach to model wave propagation in elastic waveguides. It has recently been demonstrated by the authors that a particular formulation of the scaled boundary finite element method (SBFEM) [1–3] can be applied to compute dispersion curves and mode shapes of guided waves [4–7]. The SBFEM is a very general semi-analytical method that can be utilized in a wide range of applications to model bounded or unbounded domains in frequency as well as in time domain [8–11]. Only the boundary of the computational domain is discretized in the finite element sense, while an analytical formulation is used to scale the mesh in the interior of the domain or towards infinity, respectively. Applying this idea to the comparably simple case of guided waves in infinite structures leads to the cross-section of the waveguide being discretized, while harmonic wave propagation is assumed along the waveguide [4,5]. The resulting formulation for this particular case bears similarities to the thin layer method (TLM) [12,13] as well as to the semi-analytical finite element (SAFE) method [14,15], which also require the discretization of the cross-section only but use different solution procedures. In the SBFEM, a Hamiltonian eigenvalue problem is derived for the computation of wavenumbers which can be solved very efficiently. Moreover, the computational costs are drastically reduced by utilizing spectral elements of very high order [4,5,16,17]. In a more recent development, a novel mode-tracing approach in combination with inverse iteration has been introduced to solve for the propagating modes only rather than obtaining the full set of eigenvalues [7].

In order to model a plate structure or a cylindrical waveguide, the discretization of a single straight line is sufficient to describe the structure [4,6,18]. Using a two-dimensional discretization, a cross-section of arbitrary geometry and arbitrary distribution of material parameters can be modeled [5,16]. It is this flexibility as well as the computational efficiency which make these finite element based techniques (SBFEM and SAFE but also waveguide finite elements (WFE) [19] and even full three-dimensional models [20]) very popular for the simulation of guided waves.

On the contrary, there exist a group of approaches that are based on the analytical description of the reflection and transmission of partial waves at the waveguide's interfaces. For homogeneous isotropic plates and cylinders, analytical solutions have been found long ago by Lamb [21] as well as Pochhammer [22] and Chree [23]. The most common formulation to model layered structures analytically is nowadays known as the global matrix method [24]. Most analytical approaches require the solution of a root-finding problem for the wavenumbers, which is typically supported by a mode-tracing algorithm to improve convergence and to reduce the likelihood of missing solutions. While this method is very reliable for simple structures, it can be cumbersome to obtain the complete set of solutions if the number of layers becomes large or if the attenuation of the guided waves (due to either material damping or leakage into a surrounding medium) has to be considered. In the latter case, all wavenumbers become complex which complicates the solution of the root-finding

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problem. Recently, sophisticated solution procedures based on algorithmic differentiation have been presented [25], which highly improve the reliability of the solution but entail very high computational costs. Various authors have described the modeling of simple structures embedded in fluids [26–32], where the interaction with the surrounding medium is comparably weak. Besides that, particular analytical models have been derived e.g. for the transient analysis of waves in cylinders embedded in solid media as well as waves in porous formations [33,34] and embedded concrete piles [35].

In the numerical and semi-analytical methods mentioned earlier, material damping can easily be incorporated [14], since the solution of a complex-valued eigenvalue problem is straightforward using well-established algorithms. The accurate modeling of leakage into an infinite surrounding medium on the other hand is not trivial. In fact, this has for many years been considered a major drawback of numerical methods in this fields. Probably the most obvious idea is to discretize a significant domain of the surrounding medium and attach an absorbing region to avoid reflections from the boundary of the discretized domain to interfere with the waveguide modes. The absorbing region is assumed to consist of the same material as the surrounding medium while an additional material damping – increasing with the distance to the waveguide's surface - is introduced, causing waves to vanish as they propagate towards the boundary of the discretized domain. It has been demonstrated that absorbing regions can lead to accurate results in transient finite element analyses [36–38] as well as in the SAFE method [39,40]. However, the computational costs are very high, since the dimensions of the absorbing region typically have to be chosen about 10 times larger than the waveguide [41], leading to large numbers of degrees of freedom even for simple problems [36]. Moreover, the set of solutions include a high number of unphysical modes in the absorbing region, the careful elimination of which requires additional work. As an alternative to absorbing regions, perfectly matched layers [42,43] or infinite elements [44,45] can be implemented to model the infinite medium, leading to similar advantages and drawbacks compared to absorbing regions. Specific problems arising in the application of PMLs are described in [46]. The surrounding medium can be modeled by a boundary element formulation, decoupled from the waveguide, to model the radiated sound of guided waves [47]. Recently, it has been demonstrated that a so called 2.5D boundary element formulation of the surrounding medium can be coupled with the FEM discretization of the waveguide in order to obtain a complete description of leaky guided waves [48,49]. Using this approach, the radiation condition at the waveguide's surface is satisfied exactly and the interaction of the external wave field with the waveguide is modeled. However, the coupling leads to a rather complex non-linear eigenvalue problem, the solution of which is not straight-forward.

In a recent development presented by the authors [41], an alternative approach has been suggested for the cases of embedded plate structures and cylinders. A simple dashpot boundary condition [50–53] was proposed to account for the influence of the surrounding medium. The modeling of the waveguide is formulated in terms of the scaled boundary finite element method. However, the dashpot boundary condition can be implemented similarly in other finite element based approaches. The derivation of the boundary condition is based on the assumption that each component of the displacement vector on the surface obeys the one-dimensional Helmholtz equation [54]. As a result, the surrounding medium acts as a damper with its acoustic impedances being the damping coefficients. Even though this is obviously an approximation, it was demonstrated that this technique leads to sufficiently accurate results for many practical applications. The main benefit of this approach is its simplicity regarding the implementation that leads to a simple damping term in the eigenvalue problem. No unphysical modes in the surrounding medium have to be considered and boundary element specific drawbacks like singular elements and the need for fundamental solutions are avoided. This also allows a straight-forward implementation of high-order spectral elements for the discretization of the waveguide's cross-section. Computational costs are very low, since the surrounding medium is not discretized and the solution of only a standard eigenvalue problem is required at each frequency.

In the present work, this approach is extended to include waveguides of arbitrarily shaped cross-section. The dashpot boundary condition is integrated along the boundary of the discretization, using standard finite element procedures. For verification, dispersion curves for plates and cylinders are computed that can be compared with different approaches. A more complex geometry is then presented to demonstrate the applicability to structures with arbitrary cross-section.

2. Governing equations, boundary conditions and assumptions

Consider a three-dimensional prismatic waveguide in vacuum as shown in Fig. 1. The cross-section of the waveguide is parallel to the y–z plane, whereas the system extends towards infinity in the x direction. In the case of vanishing body forces, the governing equations of three-dimensional linear elastodynamics in the frequency-domain can be expressed as [3]

$$\mathbf{L}^{\mathrm{T}}\boldsymbol{\sigma} + \omega^2\rho\mathbf{u} = 0, \tag{1}$$

with the differential operator

$$\mathbf{L} = \begin{bmatrix} \partial_x & & \\ & \partial_y & \\ & & \partial_z \\ \partial_y & \partial_x & \\ \partial_z & & \partial_x \\ & \partial_z & \partial_y \end{bmatrix}. \tag{2}$$

The stress amplitudes $\boldsymbol{\sigma}$ are related to the strain amplitudes $\boldsymbol{\varepsilon}$ through Hooke's law with the elasticity matrix $\mathbf{D}$,

$$\boldsymbol{\sigma} = \begin{bmatrix}\sigma_x & \sigma_y & \sigma_z & \tau_{xy} & \tau_{xz} & \tau_{yz}\end{bmatrix}^{\mathrm{T}} = \mathbf{D}\boldsymbol{\varepsilon}. \tag{3}$$

The strain amplitudes $\boldsymbol{\varepsilon}$ follow from the displacement amplitudes as

$$\boldsymbol{\varepsilon} = \begin{bmatrix}\varepsilon_x & \varepsilon_y & \varepsilon_z & \gamma_{xy} & \gamma_{xz} & \gamma_{yz}\end{bmatrix}^{\mathrm{T}} = \mathbf{L}\mathbf{u}. \tag{4}$$

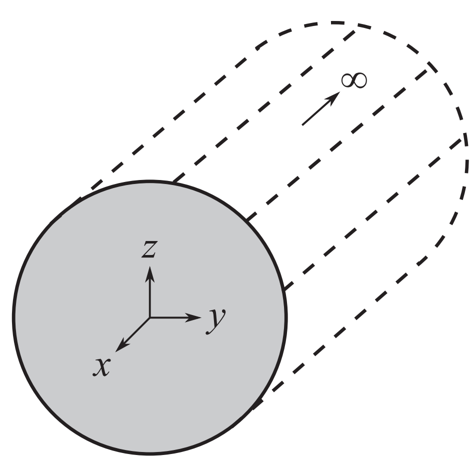

Fig. 1. Prismatic waveguide.