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Vibration reanalysis based on block combined approximations with shifting

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ABSTRACT

This paper presents an efficient method for vibration reanalysis. The procedure uses block combined approximations with shifting to simultaneously calculate some eigenpairs of modified structures. The triangular factorizations of shifted stiffness matrices generated in initial vibration analysis are utilized to calculate the higher modes. The proposed method is based on matrix–matrix operations with Level-3 BLAS and can provide very fast development of approximate solutions of high quality for frequencies and associated modal shapes of the modified structure. Numerical examples are given to demonstrate the efficiency of the vibration reanalysis algorithm and the accuracy of the approximate solutions.

1. Introduction

It is well-known that finite element analysis (FEA) is a main part of various structural design and optimization problems. In general, structures may be gradually modified during the process of design or optimization until an optimal solution satisfying the constraints is reached. Each modification needs a fresh structural analysis, and the process involves repeated and tremendous calculations.

The reanalysis methods are intended to efficiently evaluate the structural responses for various changes in design without solving the full set of modified analysis equations. The solution procedures usually use the response of the initial structure. Some vibration (eigenvalue) reanalysis methods have been proposed. One of the tools for vibration reanalysis is the matrix perturbation method. Some improved perturbation methods were presented [1,2]. These methods are efficient in case where the changes in design variables are small. To further improve the accuracy, some hybrid methods, such as Padé approximation method [3,4] and the extended Kirsch's combined method [5–7] were suggested for vibration reanalysis. The Padé approximation is used to reconstruct the basis vectors, the approximate eigenvalue is then computed by using the Rayleigh quotient. The extended Kirsch's combined method uses the first three order perturbation terms of eigenvectors as basis vectors to compute the approximate eigenvalue of the modified structure. Based on the Neumann series expansion and epsilon-algorithm, a vibration reanalysis method was developed [8]. A procedure using the iteration and inverse iteration methods with frequency-shift, and epsilon algorithm for vibration reanalysis was proposed in [9]. Massa et al. put forward a modal reanalysis method based on homotopy perturbation and projection techniques [10]. The combined approximation (CA) approach for the vibration reanalysis was developed by Kirsch [11,12]. The CA method uses binomial series terms as basis vectors in reduced basis approximations. Originally, the CA method was proposed for structural static reanalysis [13]. Afterwards, the method had been extended to vibration reanalysis. It has been shown the CA approach can provide accurate results for the lower mode shapes [14]. The basis vectors were improved by using the Gram–Schmidt orthogonalizations and shifts for linear-dynamic reanalysis [15]. A general review of reanalysis and sensitivity reanalysis by CA method was given by Kirsch [16], and the accuracy of the results and the computational cost for vibration reanalysis have been reported. The CA method, however, calculates a frequency and associated mode shape each time, it is less efficient and accurate, especially for higher order mode shapes. Modified CA methods for vibration reanalysis were proposed by Zhang et al. [17] and Mourelatos and Nikolaidis [18]. Based on the matrix inverse power iteration and the CA method, the modified CA method uses multiple basis vectors to iterate simultaneously, but no shifting strategy is used and the modified stiffness matrix must be factored for every new design.

In this paper, a block combined approximation with shifting (BCAS) is presented for the vibration reanalysis. Compared with the CA method, the proposed BCAS method is used to simultaneously compute several frequencies and associated mode shapes of the modified structure at a time. In order to improve computational efficiency, Basic Linear Algebra Subprograms (BLAS) are

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utilized. The BLAS are routines that provide standard building blocks for performing basic vector and matrix operations. They were first published as a Fortran library in 1979 [19] and are still used as a building block in higher-level mathematical programming languages and libraries, including LINPACK, LAPACK and so on. The Level-1 BLAS perform scalar, vector and vector-vector operations; the Level-2 BLAS perform matrix–vector operations; the Level-3 BLAS perform matrix–matrix operations and thereby its higher performance is achieved [20]. Because the BLAS are efficient, portable, and widely available, they are commonly used in the development of high quality linear algebra software. For vibration reanalysis, matrix–vector operations in the CA method are based on the Level-2 BLAS; while the matrix–matrix operations in the proposed BCAS method are based on the Level-3 BLAS. The execution efficiency is thus enhanced. For computing the higher eigenpairs, factorizations of the shifted stiffness matrices generated during the initial vibration analysis [21–24] are employed. As a result, factorizations of the modified stiffness matrix and associated shifted stiffness matrices are not needed. The proposed method allows very fast development of approximate solutions of high quality and low computational cost for frequencies and associated modal shapes of the modified structure. This will be demonstrated by three numerical examples of finite element models.

2. Vibration reanalysis

2.1. Problem formation

Consider an initial design with the $n \times n$ stiffness and mass matrices $\mathbf{K}_0$ and $\mathbf{M}_0$, respectively. Both of them are sparse and symmetric positive definite. The corresponding p lowest eigenpairs can be computed by solving the following eigenproblem via subspace iteration or Lanczos algorithms

$$\mathbf{K}_0\boldsymbol{\Phi}_0 = \mathbf{M}_0\boldsymbol{\Phi}_0\boldsymbol{\Lambda}_0 \tag{1}$$

where $\boldsymbol{\Lambda}_0 = \mathrm{diag}(\lambda_{01}, \lambda_{02}, \ldots, \lambda_{0p})$ and $\boldsymbol{\Phi}_0 = [\boldsymbol{\varphi}_{01}, \boldsymbol{\varphi}_{02}, \ldots, \boldsymbol{\varphi}_{0p}]$ represent the matrices of p eigenvalues and the corresponding eigenvectors of the initial structure, respectively.

It is efficient to use the shifting technique to improve the convergence rate for higher modes in the initial vibration analysis. Define the generalized eigenproblem with shifting μ

$$(\mathbf{K}_0 - \mu\mathbf{M}_0)\boldsymbol{\Phi}_0 = \mathbf{M}_0\boldsymbol{\Phi}_0\,\widehat{\boldsymbol{\Lambda}} \tag{2}$$

where $\widehat{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda}_0 - \mu\mathbf{I}$. Two eigenproblems in Eqs. (1) and (2) have the same eigenvectors. By use of subspace iteration with shifting or shifted Lanczos algorithms, the modes will converge to the ones having the smallest shifted eigenvalues. For more details, we refer readers to Refs. [21–24]. As a result, the LDL$^{\mathrm{T}}$ factorized form of initial stiffness matrix $\mathbf{K}_0$ and those of a series of shifted stiffness matrices $\mathbf{K}_0 - \mu_{0j}\mathbf{M}_0 (j = 1, \ldots, m)$ have already been given from the initial analysis as follows

$$\mathbf{K}_0 = \mathbf{L}_0\mathbf{D}_0\mathbf{L}_0^T \tag{3}$$

$$\mathbf{K}_0 - \mu_{0j}\mathbf{M}_0 = \mathbf{L}_{0j}\mathbf{D}_{0j}\mathbf{L}_{0j}^T, \quad j = 1, \ldots, m \tag{4}$$

where $\mathbf{L}_0$ and $\mathbf{L}_{0j}$ $(j = 1, \ldots, m)$ are lower triangular matrices with unit elements on their diagonals, $\mathbf{D}_0$ and $\mathbf{D}_{0j}$ $(j = 1, \ldots, m)$ are diagonal matrices, and $m + 1$ is number of these triangular factorizations.

Suppose that there are changes in the design, the corresponding changes in the stiffness and mass matrices are denoted as $\Delta\mathbf{K}$ and $\Delta\mathbf{M}$, respectively. Modified stiffness and mass matrices $\mathbf{K}$ and $\mathbf{M}$ can be written as

$$\mathbf{K} = \mathbf{K}_0 + \Delta\mathbf{K}, \quad \mathbf{M} = \mathbf{M}_0 + \Delta\mathbf{M} \tag{5}$$

The main purpose of vibration reanalysis is to efficiently obtain the approximate p eigenpairs of the modified structure without directly solving the complete eigenproblem

$$(\mathbf{K}_0 + \Delta\mathbf{K})\boldsymbol{\Phi} = \mathbf{M}\boldsymbol{\Phi}\boldsymbol{\Lambda} \tag{6}$$

where $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$ and $\boldsymbol{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_p]$ are the matrix of the requested eigenvalues and that of the corresponding eigenvectors for newly designed structure.

2.2. The CA method and the improved basis vectors

In the CA method [15], the basis vectors for each requested eigenpair $(\lambda_k, \boldsymbol{\varphi}_k)$ can be computed by the terms of the binomial series as follows. Premultiplying Eq. (6) by $\mathbf{K}_0^{-1}$, we obtain

$$(\mathbf{I} + \mathbf{B})\boldsymbol{\varphi}_k = \mathbf{r}_{0k}, \quad k = 1, \ldots, p \tag{7}$$

where

$$\mathbf{B} = \mathbf{K}_0^{-1}\Delta\mathbf{K} \tag{8}$$

$$\mathbf{r}_{0k} = \lambda_k\mathbf{K}_0^{-1}\mathbf{M}\boldsymbol{\varphi}_k \tag{9}$$

Pre-multiplying Eq. (7) by $(\mathbf{I} + \mathbf{B})^{-1}$ gives

$$\boldsymbol{\varphi}_k = (\mathbf{I} + \mathbf{B})^{-1}\mathbf{r}_{0k} \tag{10}$$

If the spectral radius $\rho(\mathbf{B})$ of matrix $\mathbf{B}$ is less than 1, use of Eq. (10) leads to the following expansion

$$\boldsymbol{\varphi}_k = (\mathbf{I} - \mathbf{B} + \mathbf{B}^2 - \cdots)\mathbf{r}_{0k} \tag{11}$$

Because $\mathbf{r}_{0k}$ is unknown, replacing $(\lambda_k, \boldsymbol{\varphi}_k)$ with $(\lambda_{0k}, \boldsymbol{\varphi}_{0k})$ in Eq. (9) results in

$$\mathbf{r}_{0k} \approx \lambda_{0k}\mathbf{K}_0^{-1}\mathbf{M}\boldsymbol{\varphi}_{0k} \tag{12}$$

Note that multiplication of a basis vector by a scalar does not affect the results [15], we can drop λ_{0k} and the first basis vector is given by

$$\mathbf{r}_1 = \mathbf{K}_0^{-1}\mathbf{M}\boldsymbol{\varphi}_{0k} \tag{13}$$

Based on Eq. (13), the basis vectors $\mathbf{r}_B = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_s]$ are calculated by the terms of the binomial series in Eq. (11)

$$\mathbf{r}_i = -\mathbf{B}\mathbf{r}_{i-1}, \quad i = 2, \ldots, s \tag{14}$$

Note that computation of each basis vector by Eqs. (13) and (14) involves only forward and backward substitutions, since $\mathbf{K}_0$ is given in the decomposed form of Eq. (3) from the initial analysis.

In summary, for computing the modified eigenpair $(\lambda_k, \boldsymbol{\varphi}_k)$, the CA method [16] involves the following five steps.

1. Compute the modified matrices $\mathbf{K} = \mathbf{K}_0 + \Delta\mathbf{K}$, $\mathbf{M} = \mathbf{M}_0 + \Delta\mathbf{M}$
2. Compute the matrix of basis vectors $\mathbf{r}_B = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_s]$ by Eqs. (13) and (14)
3. Compute the reduced matrices $\mathbf{K}_r$ and $\mathbf{M}_r$ by
$$\mathbf{K}_r = \mathbf{r}_B^T\mathbf{K}\mathbf{r}_B, \quad \mathbf{M}_r = \mathbf{r}_B^T\mathbf{M}\mathbf{r}_B \tag{15}$$
4. Solve the reduced $s \times s$ eigenproblem for the first eigenpair $(\widetilde{\lambda}_1, \mathbf{y}_1)$
$$\mathbf{K}_r\mathbf{y}_1 = \widetilde{\lambda}_1\mathbf{M}_r\mathbf{y}_1 \tag{16}$$
where
$$\mathbf{y}_1^T = \{y_1, y_2, \ldots, y_s\} \tag{17}$$
5. Approximate the requested mode shape $\boldsymbol{\varphi}_k$ by
$$\boldsymbol{\varphi}_k = y_1\mathbf{r}_1 + y_2\mathbf{r}_2 + \cdots + y_s\mathbf{r}_s = \mathbf{r}_B\mathbf{y}_1 \tag{18}$$