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# A robust version of the KPSS test based on indicators

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**Abstract**

This paper proposes a test of the null hypothesis of stationarity that is robust to the presence of fat-tailed errors. The test statistic is a modified version of the so-called KPSS statistic. The modified statistic uses the "sign" of the data minus the sample median, whereas KPSS used deviations from means. This "indicator" KPSS statistic has the same limit distribution as the standard KPSS statistic under the null, without relying on assumptions about moments, but a different limit distribution under unit root alternatives. The indicator test has lower power than standard KPSS when tails are thin, but higher power when tails are fat.


*JEL classification:* C22; C32

*Keywords:* Unit root testing; Time series; Asymptotic theory

## 1. Introduction

In this paper we wish to test the null hypothesis that an observed series $\{x_t\}$ is stationary. We allow for non-zero level (mean or median) for $x_t$, but not for deterministic trend. This is often called the hypothesis of "level stationarity," and a standard test for this hypothesis is the $\hat{\eta}_\mu$ test of Kwiatkowski et al. (1992), which we will simply call the KPSS test. For a sample $x_1, \ldots, x_T$, define $\bar{x}_T$ as the sample mean, and the demeaned data $e_t = x_t - \bar{x}_T$

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($t = 1, \ldots, T$). Then the KPSS statistic is a function of the $e_t$; the numerator is the sum of squares of the cumulations of the $e_t$, while the denominator is an estimate of their long run variance. The asymptotic distribution of the statistic is a functional of a Brownian bridge. This result depends on the series satisfying a short-memory condition and having finite variance.

In this paper we seek to relax the finite variance assumption. The motivation is an earlier paper, Amsler and Schmidt (2000), which considered the robustness (or lack of robustness) of the KPSS test to fat-tailed errors.[1] If the series follows a symmetric stable distribution with infinite variance, they showed that the KPSS statistic follows a different limit theory, based on the (demeaned) Lévy process. They also considered *local* departures from finite variance, in which case the series is assumed to be represented as follows: $x_t = x_{1t} + (c/T^{1/\alpha - 1/2})x_{2t}$, where $x_{1t}$ has finite variance and $x_{2t}$ has a symmetric stable distribution with parameter $\alpha < 2$. In this case the limit theory involves a mixture of the Wiener and Lévy processes. The asymptotic distribution depends on the "parameter" $c$ which controls the weight given to the Lévy process. Thus, in terms of asymptotics, the KPSS test is not robust to even local departures from finite variance. Their simulations correspondingly show size distortions in finite samples, not only when the data have infinite variance, but also when they have finite variance but fat tails (e.g., student's $t$ with three degrees of freedom).

In some sense it is obvious that to make the variance finite, or more generally to remove the effects of fat tails, we should trim the data. Strictly speaking, any given (not data dependent) trimming rule, like censoring the data at $\pm 100$, should work, but intuitively it seems that a sensible trimming rule should depend on the location and scale of the data. Use of a data-dependent trimming rule will raise non-trivial questions about the asymptotic theory for the statistic. In this paper we choose a rather simple trimming rule: we replace the data by an indicator that equals plus or minus one, depending on whether the observation is above or below the sample median. Using these indicators, we have handled location sensibly but have sidestepped the perhaps more difficult question of scale. We then proceed to construct the KPSS statistic in the usual fashion, but from the transformed data. We will call this test the *indicator KPSS* test.

We show that under the null, whether or not the variance is finite, so long as a short memory condition holds, we have the same asymptotic distribution as the original KPSS test has under finite variance. Thus the test is robust to infinite variance (fat tails). Under the alternative, the asymptotic distribution of the indicator KPSS test is different from that of the KPSS test, even in the finite variance case, and for both tests it is different in the finite variance case than in the infinite variance case. Thus we should expect power differences between the two tests depending on tail thickness. Our simulations show that this occurs. The indicator test is less powerful than the usual KPSS test if there are not fat tails, but it is more powerful (in terms of both power and size-adjusted power) if the tails are fat enough.

The plan of the paper is as follows. In Section 2, we provide the asymptotic theory for the indicator KPSS test. In Section 3, we report our simulations. Finally, Section 4 gives

[1]That paper also considers the modified rescaled range test of Lo (1991). We will not consider Lo's test in this paper, other than to note that it can be easily modified to be robust to fat tails in exactly the same way as we will do for KPSS.