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Testing with many weak instruments

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Abstract

This paper establishes the asymptotic distributions of the likelihood ratio (LR), Anderson–Rubin (AR), and Lagrange multiplier (LM) test statistics under "many weak IV asymptotics." These asymptotics are relevant when the number of IVs is large and the coefficients on the IVs are relatively small. The asymptotic results hold under the null and under suitable alternatives. Hence, power comparisons can be made.

Provided $k^3/n \to 0$ as $n \to \infty$, where n is the sample size and k is the number of instruments, these tests have correct asymptotic size. This holds no matter how weak the instruments are. Hence, the tests are robust to the strength of the instruments. The asymptotic power results show that the conditional LR test is more powerful asymptotically than the AR and LM tests under many weak IV asymptotics.

JEL classification: C12; C30

Keywords: Anderson–Rubin test; Conditional likelihood ratio test; Instrumental variables; Lagrange multiplier test; Many instrumental variables; Weak instruments

1. Introduction

This paper contributes to the literature on weak instrumental variables (IVs) in linear IV models. The weak IV literature documents that standard procedures, such as two-stage least squares-based t tests and confidence intervals, perform poorly when the IVs are weak

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(i.e., when the IVs are only weakly correlated with the right-hand side endogenous variables). In consequence, alternative testing procedures have been developed whose size is robust to the strength of the IVs. Such tests include the Anderson and Rubin (1949) (AR) test, the Lagrange multiplier (LM) test introduced in Kleibergen (2002) and Moreira (2001), and the conditional likelihood ratio (CLR) test introduced in Moreira (2003). Andrews et al. (2006a) have shown that the CLR test has near optimal power properties in models with Gaussian errors within a class of invariant similar tests. Furthermore, the robustness of the asymptotic size and power properties of the AR, LM, and CLR tests to non-normality has been established under the "weak IV asymptotics" of Staiger and Stock (1997), see the references above.

This paper contributes to the literature by analyzing the behavior of the AR, LM, and CLR tests when the IVs may be weak, the number of IVs, k, may be relatively large, and the equation errors may be non-normal. Specifically, the paper presents new results for these tests in the linear IV regression model under "many weak IV asymptotics" in which $k \to \infty$ as the sample size, n, goes to infinity and the strength of the IVs may be weak. Asymptotics of this type have been considered recently by Chao and Swanson (2005), Stock and Yogo (2005), Han and Phillips (2006), Anderson et al. (2005), Hansen et al. (2005), Newey and Windmeijer (2005), and Andrews and Stock (2006). Most of these papers focus on the properties of estimators. In contrast, we are interested in the properties of tests—both for testing purposes and for obtaining confidence intervals via inversion. In particular, we are interested in the properties of tests when the equation errors are non-normal.

We find that in the many weak IV asymptotic setup the CLR, AR, and LM tests are completely robust asymptotically to weak IVs with normal and non-normal errors. That is, the asymptotic levels of the tests are correct no matter how weak are the IVs. On the other hand, the asymptotic levels of the CLR, AR, and LM tests are not completely robust to the magnitude of k relative to n. One does not want to take k too large relative to n. Results of Andrews and Stock (2006) for the case of normal errors indicate that the condition $k^{3/2}/n \to 0$ as $n \to \infty$ is necessary for correct asymptotic size.[1] With non-normal errors, the results of this paper show that a sufficient condition for correct asymptotic size is $k^3/n \to 0$ as $n \to \infty$. Although this condition covers many cases of interest, it can be restrictive. For example, it is not suitable for the Angrist and Krueger (1991) example when one interacts the quarter of birth IV with state dummies to yield $k = 180$ and $n = 329,509$. Whether the condition $k^3/n \to 0$ is necessary is an open question (see the discussion below).

Andrews and Stock (2006) show that the CLR test is essentially on the asymptotic power envelope for normal errors under many weak IV asymptotics—regardless of the relative strength of the IVs to k in the asymptotics. In addition, the AR and LM tests are found not to be on the power envelope. In the present paper, we show that the asymptotic power properties of the CLR, AR, and LM tests are the same under non-normal errors as under normal errors given the $k^3/n \to 0$ condition. The aforementioned results combine to establish that the CLR test has power advantages over the AR and LM tests for non-normal as well as normal errors.

[1] This condition is necessary for the estimator of the reduced-form variance matrix to be $k^{1/2}$-consistent, and $k^{1/2}$-consistency of this estimator is necessary for the effect of estimation of the variance matrix to be asymptotically negligible.