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Bayesian analysis of a Tobit quantile regression model

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Abstract

This paper develops a Bayesian framework for Tobit quantile regression. Our approach is organized around a likelihood function that is based on the asymmetric Laplace distribution, a choice that turns out to be natural in this context. We discuss families of prior distributions on the quantile regression vector that lead to proper posterior distributions with finite moments. We show how the posterior distribution can be sampled and summarized by Markov chain Monte Carlo methods. A method for comparing alternative quantile regression models is also developed and illustrated. The techniques are illustrated with both simulated and real data. In particular, in an empirical comparison, our approach out-performed two other common classical estimators.

JEL classification: C14; C24

Keywords: Asymmetric Laplace distribution; Bayes factor; Bayesian inference; Bayesian model comparison; Markov chain Monte Carlo methods; Quantile regression; Tobit model

1. Introduction

This paper is concerned with the following problem. Suppose that y^* and y are random variables connected by the censoring relationship

$$y = \max\{y^0, y^*\},$$

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where y^0 is a known censoring point, and that we are given a sample of independent observations $\mathbf{y} = (y_1, \ldots, y_n)$ and associated covariates $\mathbf{x} = (x_1, \ldots, x_n)$, where $y_i = \max\{y_i^0, y_i^*\}$ and x_i is a k vector. The objective is to model and estimate the θth conditional quantile function of y given the sample $(\mathbf{y}, \mathbf{x})$, for $0 < \theta < 1$. Following Powell (1986) and Buchinsky and Hahn (1998), we assume that for the θth quantile, the partially latent y_i^* is generated according to the model

$$y_i^* = x_i'\beta_\theta + \varepsilon_{\theta i},$$

where the θth conditional quantile of $\varepsilon_{\theta i}$, denoted $\text{quant}_\theta(\varepsilon_{\theta i}|x_i)$, is zero. The study of this important and interesting "Tobit quantile regression" problem has been taken up by Powell (1986), Hahn (1995), Buchinsky and Hahn (1998) and Bilias et al. (2000), amongst others, and has led to a body of frequentist parametric and semiparametric methods for estimating the conditional quantile function. The purpose of this paper is to describe the first Bayesian approach for estimating a Tobit quantile regression model, extending and complementing the method developed by Chib (1992) for the standard Tobit model.

Our approach to this problem relies on the use of the asymmetric Laplace distribution as the distribution of the error $\varepsilon_{\theta i}$. As we show in Section 2, this choice is quite natural in the context of the quantile regression problem. For given families of prior distributions on the quantile regression parameter β_θ, we provide conditions under which the posterior distribution is proper. We show how appropriate Markov chain Monte Carlo (MCMC) methods can be used to simulate and summarize the posterior distribution (Tierney, 1994). Chib and Greenberg (1995) provide an excellent tutorial on Metropolis–Hastings algorithm. The approach of Chib (1995) for Gibbs output, as extended by Chib and Jeliazkov (2001) for Metropolis–Hastings chains, is also used to estimate the marginal likelihood of our model. This leads to a Bayesian framework for comparing alternative Tobit quantile regression models.

The rest of the paper is organized as follows. Assuming a Bayesian structure, in Section 2 we present details of the likelihood and prior distributions that we consider. We show that this choice leads to proper posteriors with finite moments. In Section 3 we outline the MCMC scheme that we adopt to perform the necessary Bayesian computations. We then present a series of simulation studies that illustrate the implementation of the proposed approach. The marginal likelihood which allows Bayesian Tobit model selection is derived in Section 4. The methods are applied to a real data set in Section 5. Section 6 provides additional discussion.

2. Inferential framework

Powell's (1986) estimator for the population parameter β_θ, as well as the alternative estimators proposed by Buchinsky and Hahn (1998) and Bilias et al. (2000), are based on the check (or loss) function $\rho_\theta(\lambda) = \{\theta - I(\lambda < 0)\}\lambda$, where I is the usual indicator function. An intuitive estimator for the Tobit quantile regression model is given by

$$\hat{\beta}_\theta = \arg\min_\beta \sum_{i=1}^{n} \rho_\theta(y_i - \max\{y_i^0, x_i'\beta\}). \tag{1}$$

Buchinsky and Hahn (1998) pointed out that the objective function (1) is not convex in β with the result that obtaining a global minimizer can be difficult. Instead, Buchinsky and