Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Journal of Econometrics 133 (2006) 443–477

JOURNAL OF Econometrics

www.elsevier.com/locate/jeconom

Monte Carlo tests with nuisance parameters: A general approach to finite-sample inference and nonstandard asymptotics

Jean-Marie Dufour*

Département de sciences économiques, Université de Montréal, C.P. 6128 succursale Centre-ville, Montréal, Québec, Canada H3C 3J7

Abstract

The technique of Monte Carlo (MC) tests [Dwass (1957, Annals of Mathematical Statistics 28, 181–187); Barnard (1963, Journal of the Royal Statistical Society, Series B 25, 294)] provides a simple method for building exact tests from statistics whose finite sample distribution is intractable but can be simulated (when no nuisance parameter is involved). We extend this method in two ways: first, by allowing for MC tests based on exchangeable possibly discrete test statistics; second, by generalizing it to statistics whose null distribution involves nuisance parameters [maximized MC (MMC) tests]. Simplified asymptotically justified versions of the MMC method are also proposed: these provide a simple way of improving standard asymptotics and dealing with nonstandard asymptotics.

JEL classification: C12; C15; C2; C52; C22

Keywords: Monte Carlo test; Maximized Monte Carlo test; Finite-sample test; Exact test; Nuisance parameter; Bounds; Bootstrap; Parametric bootstrap; Simulated annealing; Asymptotics; Nonstandard asymptotic distribution

---

*Tel.: + 1 514 343 2400; fax: + 1 514 343 5831.

E-mail address: jean.marie.dufour@umontreal.ca.

URL: http://www.fas.umontreal.ca/SCECO/Dufour.

doi:10.1016/j.jeconom.2005.06.007

1. Introduction

During the last 25 years, the development of faster and cheaper computers has made Monte Carlo techniques more affordable and attractive in statistical analysis. In particular, such techniques may now be used routinely for data analysis. Important developments in this area include the use of bootstrap techniques for improving standard asymptotic approximations (for reviews, see Efron, 1982; Beran and Ducharme, 1991; Efron and Tibshirani, 1993; Hall, 1992; Jeong and Maddala, 1993; Vinod, 1993; Shao and Tu, 1995; Davison and Hinkley, 1997; Chernick, 1999; Horowitz, 1997) and techniques where estimators and forecasts are obtained from criteria evaluated by simulation (see McFadden, 1989; Mariano and Brown, 1993; Hajivassiliou, 1993; Keane, 1993; Gouriéroux and Monfort, 1996; Gallant and Tauchen, 1996).

With respect to tests and confidence sets, these techniques only have asymptotic justifications and do not yield inferences that are provably valid (in the sense of correct levels) in finite samples. Here, it is of interest to note that the use of simulation in the execution of tests was suggested much earlier than recent bootstrap and simulation-based techniques. For example, randomized tests have been proposed long ago as a way of obtaining tests with any given level from statistics with discrete distributions (e.g., sign and rank tests); see Lehmann (1986). A second interesting possibility is the technique of Monte Carlo tests originally suggested by Dwass (1957) for implementing permutation tests and later extended by Barnard (1963), Hope (1968) and Birnbaum (1974). This technique has the great attraction of providing *exact* (randomized) tests based on any statistic whose finite-sample distribution may be intractable but can be simulated. The validity of the tests so obtained does not depend at all on the number of replications made (which can be small). Only the power of the procedure is influenced by the number of replications, but the power gains associated with lengthy simulations are typically rather small. For further discussion of Monte Carlo tests, see Besag and Diggle (1977), Dufour and Kiviet (1996, 1998), Edgington (1980), Edwards (1985), Edwards and Berry (1987), Foutz (1980), Jöckel (1986), Kiviet and Dufour (1997), Marriott (1979) and Ripley (1981).

An important limitation of the technique of Monte Carlo tests is the fact that one needs to have a statistic whose distribution does not depend on nuisance parameters. This obviously limits considerably its applicability. The main objective of this paper is to extend the technique of Monte Carlo tests in order to allow for the presence of nuisance parameters in the null distribution of the test statistic.

In Section 2, we summarize and extend results on Monte Carlo (MC) tests when the null distribution of a test statistic does not involve nuisance parameters. In particular, we put them in a form that will make their extension to cases with nuisance parameters easy and intuitive, and we generalize them by allowing for MC tests based on exchangeable (possibly nonindependent) replications and statistics with discrete distributions. These generalizations allow, in particular, for various nonparametric tests (e.g., permutation tests) as well as test statistics where certain