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# MMC techniques for limited dependent variables models: Implementation by the branch-and-bound algorithm

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**Abstract**

We propose a finite sample approach to some of the most common limited dependent variables models. The method rests on the maximized Monte Carlo (MMC) test technique proposed by Dufour [1998. Monte Carlo tests with nuisance parameters: a general approach to finite-sample inference and nonstandard asymptotics. Journal of Econometrics, this issue]. We provide a general way for implementing tests and confidence regions. We show that the decision rule associated with a MMC test may be written as a Mixed Integer Programming problem. The branch-and-bound algorithm yields a global maximum in finite time. An appropriate choice of the statistic yields a consistent test, while fulfilling the level constraint for any sample size. The technique is illustrated with numerical data for the logit model.


*JEL classification:* C12; C15; C24; C25; C44

*Keywords:* Finite sample inference; Randomized tests; Maximized Monte Carlo tests; Branch-and-bound algorithm; Limited dependent variables model

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## 1. Introduction

Most currently available econometric procedures are based on asymptotic approximations. Their reliability and properties in finite samples are then often examined by simulations. Such simulations typically involve a large number of computations, especially in presence of nuisance parameters. Moreover, it could even be claimed that the size of the tests and confidence regions, or estimator's biases are never controlled, since the number of simulations is always finite. While simulations can never assert that a given asymptotic inference method has a satisfactory behavior, they may be used to prove that this method performs poorly. The econometric and statistical literature contains plenty of such illustrations (see for instance Davidson and MacKinnon, 1983, 1984; Orme, 1990; Savin and Würtz, 1999 for dichotomous models, Laroque and Salanié, 1994 for the disequilibrium model).

The reasons why most practitioners prefer asymptotic econometrics are many. At first glance, one may claim that the sample is "large enough" so that asymptotic approximations ought to be "reasonably good". Since, as explained above, finite sample behaviors are rarely known nor accurately studied, this is merely an act of faith. A more pragmatic justification arises from the consideration of the available exact procedures.

On the one hand, and after many years of disputes, the "Bayesian choice" remains controversial. Its opponents stress the subjective nature of the prior's choice, and/or the difficulties to control the numerical accuracy of the integration procedures (see Casella and Robert, 2004). On the other hand, "prior-free" approaches are often ad hoc. Indeed, they exploit specific features of the model and/or the inference problem to obtain pivotal distributions by means of principles such as conditioning, invariance, etc. However, a distributional property obtained for a particular model is usually difficult to adapt to another one.

A technique first proposed by Dwass (1957) and recently generalized by Dufour (this issue) considerably extends the scope of finite sample inference methods. The only requirement is that the null distribution of the test statistic on which the inference is based could be resampled from.

Dwass and Dufour use this technique to provide exact tests. The original argument by Dwass (1957) applies to a simple null hypothesis, whereas the important extension by Dufour (this issue) covers the case where nuisance parameters are present under the null. The main argument may easily be described for point null hypotheses. Assume the data generating process is represented by some given probability $\overline{\mathbb{P}}$. A point null hypothesis takes the form $\overline{\mathbb{P}} = \mathbb{P}_0$. Further assume that we are able to draw one sample from $\mathbb{P}_0$, independently from the observed data. Next consider a given statistic $T()$ which may be computed from the original sample (we denote it $T(\overline{\mathbb{P}})$) and from the simulated sample: $T(\mathbb{P}_0)$.[1] Under the null hypothesis, the probabilities $\overline{\mathbb{P}}$ and $\mathbb{P}_0$ are equal, thus the couple $(T(\overline{\mathbb{P}}), T(\mathbb{P}_0))$ forms an exchangeable process of size two. This implies that the decision rule "Reject the null hypothesis whenever $T(\overline{\mathbb{P}}) > T(\mathbb{P}_0)$" is a test at level 50%.[2] Indeed,

[1]Both samples must have the same size.

[2]Assuming $\mathbb{P}_0(T(\overline{\mathbb{P}}) = T(\mathbb{P}_0)) = 0$.