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Structural reliability analysis using a copula-function-based evidence theory model

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ABSTRACT

Evidence theory contains powerful features for uncertainty analysis and can be effectively employed to address the epistemic uncertainty, which is attributed to a lack of information in complex engineering problems. This paper presents an evidence theory model based on the copula function and the related structural reliability analysis method. It is an effective tool for uncertainty modeling and reliability analysis with dependent evidence variables. In the evidence theory model, a canonical maximum likelihood (CML) method was adopted to estimate the correlation parameter, and the Akaike information criterion (AIC) was utilized to select a reasonable Archimedean copula function and whereby construct the joint basic probability assignment (BPA) for the multidimensional evidence variables. Based on the joint BPA function, a procedure for reliability analysis was formulated to compute the reliability interval on the structure with evidence uncertainty. Four numerical examples were provided to verify the effectiveness of the proposed method.

1. Introduction

In engineering practice, uncertainties related to material characteristics, geometrical sizes and boundary conditions widely exist. The understanding, quantification and control of various uncertainties can significantly impact reliability design and comprehensive performance of structures and products [1]. Helton [2] divided uncertainties into two different kinds: aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty (also called stochastic uncertainty or objective uncertainty) results from the intrinsic variation of the system or the environment, and the probability theory is usually used to analyze this kind of uncertainty. Based on the probability theory, many traditional reliability analysis methods have been well established, like the first order reliability method (FORM) [3,4], second order reliability method (SORM) [5,6], response surface methodology [7], Monte Carlo simulations [8], system reliability method [9,10], reliability-based design optimization (RBDO) [11–14], and engineering application of reliability analysis [15,16]. Epistemic uncertainty, which is also named subjective and reducible uncertainty, stems from a lack of knowledge or data. And the typical methods to quantify epistemic uncertainty are possibility theory [17,18], fuzzy sets [19], convex models [20–26], evidence theory [27–31] and so on. Compared with the other ones, Evidence theory seems a more flexible method for treating of the epistemic uncertainty.

Evidence theory is a type of analysis method that addresses uncertainty problems by conforming with a person's thought processes and can provide reasonable depictions of incomplete, unreliable or conflicting information. Evidence theory is a generalized model of the existing uncertainty analysis theories. Under different cases, it will be equivalent to classical probability theory, possibility theory, fuzzy sets theory and convex model theory. Due to its powerful ability for epistemic uncertainty modeling, evidence theory has been receiving more and more attention in the field of structural reliability analysis. Tonon et al. [32] applied evidence theory to resolve uncertainty analysis problems in rock engineering and implemented reliability designs for tunnels. Oberkampf and Helton [33] used a simple algebra function to explore the application of evidence theory in engineering reliability and summarized the strengths and weaknesses of its application. Agarwal et al. [34] proposed the application of evidence theory based on the trust region method to address multidisciplinary optimization problems in the reliability design. By establishing a multipoint approximation for the limit-state function, Bae et al. [35,36] proposed a computational method to efficiently solve the structural reliability problem with evidence variables. Mourelatos and Zhou [37] proposed an evidence-based design optimization (EBDO) that could quickly search for the optimum point. By combining evidence theory and the traditional probability model, Du [38] created

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a hybrid reliability analysis model that could solve co-existing random uncertainty and epistemic uncertainty and furthermore proposed a sensitivity analysis method based on this model [1]. Guo et al. [39] proposed a reliability optimization design method based on evidence theory and interval analysis, and employed the model in pressure vessel design. Bai et al. [40] proposed the concept of "moments of evidence theory" and applied the concept in structural static and dynamic uncertainty analysis.

Although some progresses have been made, structural reliability analysis based on evidence theory remains preliminary and some key technical problems remain unresolved; reliability analysis with parameter correlation is one such problem. Existing reliability analysis of evidence theory is primarily aimed at the problems with independent variables, namely, an assumption that all the evidence variables are independent to each other generally needs to be made. However, parameter dependencies intrinsically exist in most systems [41] and often have profound implications for risk assessments or even dominate system performance, such as nonlinearity in physical systems. To develop an evidence-theory-based structural reliability analysis for problems with dependent parameters thus seems necessary.

Recently, as an efficient mathematic tool to describe the dependence between random variables, copula function is widely used. Copula function is a link between marginal cumulative distribution functions (CDFs) and joint CDF [42], and it allows the choice of joint CDF to be separate from the marginal CDFs. Extensive applications of copula function can be found in financial and hydrological disciplines [42–44], and in reliability analysis field, there are also some works using copula function. Lebrun and Dutfoy [45] pointed out that the commonly used Nataf transformation in traditional reliability analysis was equivalent to the Gaussian copula function. Noh et al. [46] made a comparison between two selection approaches of copula functions, namely Goodness of Fit (GOF) test and Bayesian method, and furthermore used the copula function to handle RBDO problem [47]. Tang et al. [48] analyzed the influences of different bivariate copula functions to reliability analysis results, and subsequently used the copula functions to conduct system reliability analysis [49]. However, the above applications of copula functions are all limited to aleatory uncertainty problems, and few work so far could be found on epistemic uncertainty problem.

This paper suggests a new uncertainty analysis model based on evidence theory and corresponding reliability analysis method. The contributions of this paper are twofold. First, through introducing the copula function, the suggested evidence theory model can effectively deal with the correlations existing in the evidence variables, and hence significantly expand the applicability of evidence theory in structural uncertainty analysis. Second, we formulate a reliability analysis method based on the new evidence theory model, which could greatly reduce the analysis errors of the conventional evidence-theory-based reliability analysis methods based on the assumption that all the variables are independent to each other. The remainder of the paper is organized as follows: Section 2 introduces the basic principles of evidence theory; Section 3 proposes the copula-function-based evidence theory model; Section 4 applies the evidence theory model to structural reliability analysis; Section 5 details four numerical examples to verify the effectiveness of the proposed method; Section 6 summarizes the entire paper.

2. Basic principles of evidence theory

Because evidence theory was initially proposed by Dempster and Shafer, it is also named Dempster–Shafer theory [27]. Some main concepts of evidence theory are described as follows:

(1) *Frame of discernment (FD)*. For a problem that contains uncertainty, the likelihood of the occurred events will generally take some possible sets that may be either nested within one another or partially overlapped. The FD is defined as the finest possible subdivisions of the sets, which is named the elementary proposition. The FD is similar to the finite sample space of a random variable in probability theory and is composed of all finite elementary propositions. For example [34], if FD is given as $X=\{x_1,\ x_2\}$, we have two mutually exclusive elementary propositions x_1 and x_2. All possible subset propositions of X will form a power set 2^X; in the above example, $2^X=\{\varnothing,\ \{x_1\},\ \{x_2\},\ \{x_1,\ x_2\}\}$. In this paper, we use X to denote either an evidence variable or an FD.

(2) *Basic probability assignment (BPA)*. As an important concept of evidence theory, the BPA can be used to describe the trustworthiness of a proposition. Assuming that Θ is the FD, if the mapping function m: $2^X \to [0,1]$ (2^X is Θ's power set) satisfies the following three properties,

$$m(A) \geqslant 0, \quad \forall A \in 2^X \tag{1}$$

$$m(\varnothing) = 0 \tag{2}$$

$$\sum_{A\in 2^X} m(A) = 1 \tag{3}$$

then m is named the BPA on FD Θ. $\forall A \in 2^X$, $m(A)$ is the mass of A, where set A with $m(A) > 0$ is named the focal element of m and $m(A)$ reflects the degree of support from the evidence toward the proposition in which a certain element of X belongs to set A or is the degree of believing proposition A by the decision maker under such evidence, which is similar to the probability density function of a random variable.

(3) *Evidence combining rule*. The evidences provided by different experts need to be combined to construct the total degree of support for a proposition. Assuming that m_1 and m_2 are two BPA functions under the same FD Θ and the focus elements are $B_1,B_2,\ldots,B_k$ and $C_1,C_2,\ldots,C_r$, then the combined BPA function is expressed as,

$$m(A) = \begin{cases} 0 & A = \varnothing \\ \frac{\sum_{B_i\cap C_j = A} m_1(B_i)m_2(C_j)}{1-K} & A \neq \varnothing \end{cases} \tag{4}$$

where

$$K = \sum_{B_i\cap C_j=\varnothing} m_1(B_i)m_2(C_j) \tag{5}$$

K depicts the degree of conflicts among evidences from different experts. A large K value indicates a more intense disagreement.

(4) *Belief measure (Bel) and plausibility measure (Pl)*. Due to a lack of information, the use of an interval to describe the degree of truth for a proposition is more reasonable. Evidence theory uses $\mathrm{Bel}(A)$ and $\mathrm{Pl}(A)$ to describe the degree of truth for proposition A as,

$$\mathrm{Bel}(A) = \sum_{C\subset A} m(C) \tag{6}$$

$$\mathrm{Pl}(A) = \sum_{C\cap A\neq\varnothing} m(C) \tag{7}$$

where $\mathrm{Bel}(A)$ is the sum of BPAs for all evidences that fully support proposition A and $\mathrm{Pl}(A)$ is the sum of all BPAs for all evidences that fully or partially support proposition A. One measure is the lower bound, and the second measure is the upper bound. Both of these measures form an interval of upper and lower probabilities to describe the uncertainty