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Alternative bootstrap procedures for testing cointegration in fractionally integrated processes ☆

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Abstract

This paper considers alternative methods of testing cointegration in fractionally integrated processes, using the bootstrap. The investigation focuses on (a) choice of statistic, (b) use of bias correction techniques, and (c) designing the simulation of the null hypothesis. Three residual-based tests are considered, two of the null hypothesis of non-cointegration, the third of the null hypothesis that cointegration exists. The tests are compared in Monte Carlo experiments to throw light on the relative roles of issues (a)–(c) in test performance.

JEL classification: C22

Keywords: Fractional cointegration; Bootstrap tests

1. Introduction

Except in a few simple situations, tests in econometrics are performed by the method of first-order asymptotic approximation, employing a test statistic whose asymptotic distribution under the null hypothesis is free of nuisance parameters, and

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so can be tabulated. Such statistics are said to be asymptotically pivotal. They typically depend on nuisance parameters, even though their asymptotic distributions do not, and these must be replaced by consistent estimates. The true rejection probability of the test accordingly differs from the nominal probability, generally by an error of order $T^{-1/2}$ where T is the sample size. Following Davidson and MacKinnon (1999) we refer to this error in rejection probability as the ERP.

In the bootstrap procedure, Monte Carlo simulation is used to estimate the null distribution of a test statistic directly. In this case, estimated parameters are used to construct the data generation process (DGP) from which the replications are drawn.[1] As is well known (among many references, see Hall (1992) and Horowitz (1997, 2001) for a survey) bootstrap tests based on asymptotically pivotal statistics may achieve an ERP of smaller order in T than first-order asymptotic tests, yielding the so-called bootstrap refinements. However, the bootstrap method can also be used to derive tests based on non-asymptotically pivotal statistics, allowing consideration of procedures having no counterpart in the usual armoury of asymptotic tests. The potential benefits of this approach are clear. New tests might be constructed, on an ad hoc basis and with minimal theoretical labour, with a view to maximizing power against alternatives of interest. The strong assumptions needed to justify particular established procedures might often be dispensed with. Cointegration tests provide an excellent illustration of these points. There also exist bias reduction techniques for bootstrap tests such as prepivoting (Beran, 1988), that can confer some of the advantages of asymptotically pivotal tests on the general case.

In this paper, bootstrap methods for tests of cointegration in time series that are fractionally integrated are proposed and compared. In the terminology of Granger (1986), a collection of time series are said to be $\mathrm{CI}(d,b)$ if the series are fractionally integrated of order d (or $I(d)$) while there exists a linear combination of the series which is $I(d-b)$ for $b>0$. A difficulty with this testing problem is that the distributions of the usual test statistics, under the null hypothesis $b=0$, depend on nuisance parameters asymptotically. While it is possible to devise an asymptotically pivotal test for this case based on the Johansen (1988, 1991) procedure (see Breitung and Hassler, 2002) the focus of attention here is on simple residual based statistics, that are not asymptotically pivotal.

Section 2 of the paper describes the fractional cointegration set-up, and considers a number of possible tests, both of the hypothesis of non-cointegration and of the hypothesis of cointegration, i.e., of $I(0)$ cointegrating residuals. Section 3 reviews the bootstrap procedure itself, including the 'double bootstrap' and 'fast-double bootstrap' variants. Section 4 reports the results of Monte Carlo experiments to investigate the properties of the tests. Section 5 gives an application of the procedures to US quarterly consumption, income and wealth data. Section 6 contains concluding remarks. The appendices contain proofs, and some supplementary results on the asymptotics of the cointegration test applied to fractional processes.

[1] In this context the expression DGP is used specifically to refer to a computer algorithm that generates a series whose joint distribution is intended to mimic that of an observed sample.