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Reliability based topology optimization for continuum structures with local failure constraints

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ABSTRACT

This paper presents an effective method for stress constrained topology optimization problems under load and material uncertainties. Based on the Performance Measure Approach (PMA), the optimization problem is formulated as to minimize the objective function under a large number of (stress-related) target performance constraints. In order to overcome the stress singularity phenomenon caused by the combined stress and reliability constraints, a reduction strategy on target reliability index is proposed and utilized together with the ε-relaxation approach. Meanwhile, an enhanced aggregation method is employed to aggregate the selected active constraints using a general K–S function, which avoids expensive computational cost from the large-scale nature of local failure constraints. Several numerical examples are given to demonstrate the validity of the present method.

1. Introduction

Uncertainties in external loads and material properties may have considerable impacts on the optimal structural layout. Over the past decade, reliability based topology optimization (RBTO) [1,2], which incorporates appropriate uncertainty evaluation methods into the topology optimization process, has been received increasing attention in the structural optimization community. In order to present an integrated optimal layout accounting for probabilistic or non-probabilistic uncertainties, various methods of RBTO have been developed. In the work of [3], Kim et al. performed the RBTO by using evolutionary structural optimization (ESO) combined with the advanced first order reliability method. Eom et al. [4] presented a reliability based topology optimization for 3-D structures based on bi-directional evolutionary structural optimization (BESO) and a standard response surface method. Silva et al. [5] adopted the single loop method to eliminate the inner reliability analysis loop in RBTO. By applying a projection method with erosion and dilation operators, Wang et al. [6] and Schevenels et al. [7] proposed a robust topology optimization method to simulate the effect of uniform and non-uniform manufacturing errors. Besides, non-probability reliability was studies within structural topology optimization problems by Luo et al. [8]. A level-set based shape and topology optimization under geometric uncertainty was also investigated by Chen and Chen [9]. Moreover, Lazarov et al. [10] introduced the stochastic collocation methods in topology optimization for uncertain mechanical systems. In addition, RBTO has been successfully applied to the conceptual design of many practical and multidisciplinary problems, including convection heat transfer systems [11], geometrically nonlinear structures [12], MEMS mechanisms [13–15], and electro-thermal-compliant mechanisms [16].

Although the theory of RBTO has been well investigated, the majority of existing applications focuses on stiffness-maximization related problems. Only a few studies have addressed the issue of possible strength failure of material in RBTO problems. For example, Patel and Choi [17] solved RBTO problems with stress constraints by using a classification based surrogate modelling approach. In fact, structural strength (or stress) is usually the most important design criterion from the viewpoint of structural safety.

In terms of stress constrained topology optimization with SIMP [18] approach, a difficulty is the so-called "stress singularity phenomenon", which has been studied in both truss [19] and continuum optimization problems [20]. For a deterministic optimization problem, this difficulty can be successfully overcome by the "ε-relaxation" [21] or the "qp relaxation" approach [22]. However, as will be shown in this study, incorporating uncertainties into stress constraints makes the singularity phenomenon even more challenging, especially for high-reliability requirements. It is

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because the satisfaction of reliability constraints is not only determined by the actual values of the material stresses, but also the level of target reliability. As a consequence, the connectivity of the feasible domain may not be ensured, which would hinder the effectiveness of gradient based optimal searching. To tackle this difficulty, a new numerical technique is highly desired.

This paper aims at developing an efficient RBTO scheme for stress-constrained problems of continua. The considered uncertainties include external load magnitude and material parameters. By using the performance measure approach, the RBTO problem is formulated as to minimize the objective function under target performance constraints on all elements. The main novelty is in the introduction of the reduction on target reliability index for eliminating the stress singularity phenomenon in a disconnected feasible domain. The necessity of such technique is validated through a classical truss example. Furthermore, in order to efficiently solve this nested optimization problem, an explicit iterative manner is used to search target performance points. Meanwhile, an enhanced aggregation method is adopted to handle large-scale constraints. Several numerical examples are presented to show the effectiveness of this method.

2. Reliability based stress-constrained problem

2.1. Optimization problem formulation

Using the probabilistic definition of the random uncertainties, a common formulation of RBTO for the stress-constrained problem is expressed in finite element form as follows:
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$$\begin{aligned} \min_{\rho} \quad & V=\sum_{e=1}^{N}\rho_e V_e \\ \text{s.t.} \quad & \mathbf{K}(\boldsymbol{\rho})\mathbf{u}=\mathbf{f} \\ & P_r[f_e(\mathbf{X})=g_e(\sigma_e,\mathbf{X})-1\leqslant 0]\geqslant R_{e,\text{target}}\ (e=1,2,\ldots,N) \\ & 0<\rho_{\min}\leqslant\rho_e\leqslant 1 \qquad (e=1,2,\ldots,N) \end{aligned} \tag{1}$$

where design variable vector $\boldsymbol{\rho}=[\rho_1,\rho_2,\ldots,\rho_N]^{\mathrm{T}}$ represents the relative density values of elements, V is the total material volume, N is the total number of finite elements and V_e represents the element volume. $\mathbf{K}(\boldsymbol{\rho})\mathbf{u}=\mathbf{f}$ refers to the equilibrium equation, where $\mathbf{K}$, $\mathbf{u}$ and $\mathbf{f}$ are the global stiffness matrix, the displacement vector and the load vector, respectively. $\mathbf{X}$ is the vector of random parameters of loads and material properties, $P_r[\cdot]$ is the probability of the random event and $R_{e,\text{target}}$ is the specified target probability of structural reliability. The function $f_e(\mathbf{X})$ is known as the limit state function. $\boldsymbol{\sigma}_e=[\sigma_{xx}\quad\sigma_{yy}\quad\sigma_{zz}\quad\sigma_{xy}\quad\sigma_{yz}\quad\sigma_{zx}]^{\mathrm{T}}$ denotes the average stress of the e-th element and $g_e(\boldsymbol{\sigma}_e,\mathbf{X})-1$ is the dimensionless stress constraint function. $\rho_{\min}=10^{-2}$ is set as the lower limit of the design variables. The global stiffness matrix $\mathbf{K}(\boldsymbol{\rho})$ is assembled from the elemental stiffness $\mathbf{K}_e$ as according to the rule of the SIMP approach [23,24] as $\mathbf{K}(\boldsymbol{\rho})=\sum_{e=1}^{N}\rho_e^p\mathbf{K}_e$. In this study, the penalization factor $p=3$ is chosen.

It should be noted that the dimensionless stress constraint $g_e-1\leqslant 0$ depends on the material failure behavior as well as the adopted failure criterion. Typically, when a pressure-independent material (e.g. most of the metallic materials) is used, it can be expressed from the von Mises failure criterion as:

$$g_e=\sqrt{(\boldsymbol{\sigma}_e)^{\mathrm{T}}\mathbf{V}(\boldsymbol{\sigma}_e)}\Big/\omega \tag{2}$$

For some of the nonmetallic materials, such as concrete, rocks and soils, the Drucker–Prager (D–P) failure criterion may be considered to identify the failure surface, such that the stress constraint function is given by:

$$g_e=\left(\alpha\mathbf{w}\boldsymbol{\sigma}_e+\sqrt{(\boldsymbol{\sigma}_e)^{\mathrm{T}}\mathbf{V}(\boldsymbol{\sigma}_e)}\right)\Big/\omega \tag{3}$$

where α and ω are material parameters. The vector $\mathbf{w}$ and the material $\mathbf{V}$ are constants. For a plane stress state, the following expressions hold:

$$\mathbf{w}=[1\quad 1\quad 0];\ \mathbf{V}=\begin{bmatrix}1 & -1/2 & 0\\ -1/2 & 1 & 0\\ 0 & 0 & 3\end{bmatrix} \tag{4}$$

After transforming the random variables $\mathbf{X}$ to a set of independent standard normal random ones U via the Rosenblatt transformation [25], the failure surface $f_e(\mathbf{X})=0$ in the original space is mapped into the corresponding failure surface $f_e(\mathbf{U})=0$ in the normalized space (U-space). Reliability constraints in (1) is conveniently represented in terms of a safety-index as:

$$\beta[f_e(\mathbf{U})\leqslant 0]\geqslant\beta_{e,\text{target}}\quad(e=1,2,\ldots,N) \tag{5}$$

where $\beta=\min_{U\in\{f_e(U)=0\}}\left(\sqrt{\mathbf{U}^{\mathrm{T}}\mathbf{U}}\right)$ is the Hasofer and Lind safety-index [26], $\beta_{e,\text{target}}$ is the target reliability index for the e-th constraint.

The target reliability index $\beta_{e,\text{target}}$ can be approximately determined using some probability integration methods, such as the First-Order Reliability Method (FORM) or the Second-Order Reliability Method (SORM). In FORM, a linear approximation of the performance function is used and thus $\beta_{e,\text{target}}=\Phi^{-1}(R_{e,\text{target}})$, where $\Phi(\cdot)$ is the standard Gaussian cumulative function. Generally speaking, the FORM estimate provides adequate accuracy for most practical circumstances and has been widely for reliability-based optimization applications. In SORM, the second-order sensitivities are required and the performance function is approximated as a quadratic surface to the m-dimensional random vector U. As suggested by Breitung [27], the SORM-based probability can be calculated using the theory of asymptotic approximations and rotationally invariant measure, that is

$$P_r[f_e\leqslant 0]\approx 1-\Phi(-\beta)\prod_{l=1}^{m-1}(1-\beta\kappa_l)^{-1/2}=\Psi(\beta) \tag{6}$$

where κ_l denotes the principal curvatures of the limit state function at the minimum distance point. Correspondingly, the target reliability index is calculated by $\beta_{e,\text{target}}=\Psi^{-1}(R_{e,\text{target}})$. In addition, dimension-reduction method (DRM) [28] has also been developed for the replacement of probability constraints by reliability index constraints in the reliability-based optimization of nonlinear systems by e.g. Lee et al. [29]. In this study, only the FORM approximations are used since the considered stress constraints functions would be linear or mildly nonlinear with respect to the uncertain quantities within their main variation bounds.

Based on the Performance Measure Approach (PMA) [30–32], the design problem (1) can be transformed into its equivalent optimization problem

$$\begin{aligned} \min_{\rho} \quad & V=\sum_{e=1}^{N}\rho_e V_e \\ \text{s.t.} \quad & \mathbf{K}(\rho)\mathbf{u}=\mathbf{f} \\ & \alpha_e(\boldsymbol{\sigma}_e)\leqslant 0 \qquad (e=1,2,\ldots,N) \\ & 0<\rho_{\min}\leqslant\rho_e\leqslant 1 \quad (e=1,2,\ldots,N) \end{aligned} \tag{7}$$

where $\alpha_e(\boldsymbol{\sigma}_e)$ is the performance measure value corresponding to the e-th reliability constraint with the prescribed target reliability index $\beta_{e,\text{target}}$. That is

$$\begin{aligned} \alpha_e(\boldsymbol{\sigma}_e) &= \max_{\mathbf{U}} f_e(\mathbf{U}) \\ \text{s.t.}\quad & \mathbf{U}^{\mathrm{T}}\mathbf{U}\leqslant\beta_{e,\text{target}}^2 \end{aligned} \tag{8}$$