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Limit analysis of 3D masonry block structures with non-associative frictional joints using cone programming

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ABSTRACT

A three-dimensional limit analysis model for masonry structures is presented. In the model the masonry is discretized as an assemblage of rigid blocks, which interact via no-tension contact surfaces with Coulomb friction. A concave contact formulation is adopted and an iterative solution procedure is used to allow the underlying non-associative friction problem to be solved. Second order cone programming (SOCP) is used to allow direct modelling of the conic failure surface. The formulation is validated against various numerical benchmark problems and then successfully applied to masonry walls and a small-scale masonry building tested experimentally.

1. Introduction

The collapse of dry-jointed masonry structures can be analysed using a rigid block limit analysis modelling strategy. When using such a strategy, masonry walls can be discretized into a number of rigid bodies which interact through dry contact interfaces. At such interfaces failure (or 'yield') can be permitted to occur and, as shown by pioneering workers such as Kooharian [1], Drucker [2] and Heyman [3], plastic limit analysis can be used to directly establish the collapse load and corresponding failure mechanism, without the need to consider loading history. As pointed out by Livesley [4], in such circumstances mathematical programming can conveniently be used to obtain a solution, giving the load factor and failure mechanism at incipient collapse.

The form of the mathematical programming formulation depends largely on the form of the chosen failure criteria and plastic flow rule. According to the definition given in [5], either 'convex' or 'concave' formulations can be used, depending on the number of contact points and the generalized stress components adopted in order to model the interactions between blocks. Fig. 1, which highlights a representative contact interface in a stretcher bonded (or 'running bond') wall, shows both concave and convex formulations. (Note that, with such a bonding pattern, the blocks in each successive course are staggered by half a block; this is assumed throughout the present study.)

In the case of a convex contact interface formulation (sometimes referred to as a 'surface' contact formulation), the normal and tangential stresses acting at an interface are represented by stress resultants acting at a single point, generally located at the centre of the interface; this governs the behaviour of the entire interface. The stress resultant vector is composed of six components, including normal and shear forces and bending and torsional actions, corresponding in a virtual work sense to the internal degrees of freedom at the contact interface. The use of this formulation might appear to imply that the two surfaces are in contact at a single point, and that the two surfaces are slightly convex, but the bending and torsional components take into account interactions along the entire surface [5].

In the case of a concave contact interface formulation (sometimes referred to as a 'point' contact formulation), it is assumed that the blocks interact via a number of contact points, usually located at the corners of the interface, thus appearing to imply that the two surfaces in contact are slightly concave [5]. The stress resultants acting on each contact point consist of a normal force and two shear force components.

When the blocks are assumed to possess infinite compressive strength, failure generally involves separation, rocking, sliding, or twisting at the contact interfaces, or combinations of these modes of deformation. The contact behaviour is governed by the prescribed failure criteria, expressed in terms of static variables as

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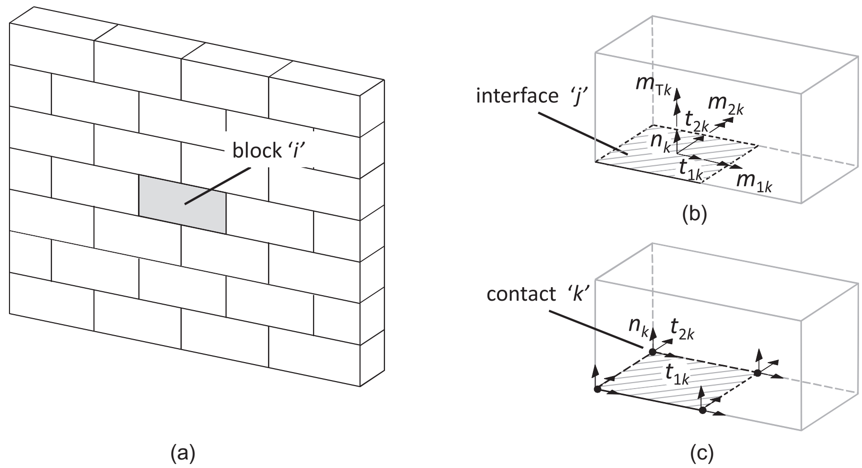

Fig. 1. (a) Three-dimensional rigid block assemblage; Contact points 'k' and static variables for block 'i' and interface 'j' in: (b) convex (or 'surface'), and (c) concave (or 'point') contact formulations.

well as by flow rules which define the relationship between displacement rates and plastic flow multipliers. Although the use of convex contacts generally involves a reduced number of contact relations per block interface, the adoption of a concave contact formulation normally leads to a simpler limit analysis problem formulation. This is because the use of a convex contact interface model generally requires several potential failure conditions to be considered (i.e. to define failure due to pure bending, sliding and torsion), and, to properly describe potential three-dimensional responses, also the interaction between these. This leads to nonlinearities in the equations governing failure and plastic flow [6]. In contrast, when a concave formulation is used the relations involved are much simpler [5,7]. This is because behaviour at contact points is governed by opening and sliding conditions only, with bending and torsion strength of an entire contact interface being governed by the combined effect of interactions at all constituent contact points. As a consequence, no specific failure criterion has to be defined to model torsion failure or interaction effects. In addition, in the case of the concave formulation it will be shown that the limit analysis problem can be directly cast as a second order cone programming (SOCP) problem, for which efficient solution algorithms exist. In contrast, in the case of the convex formulation, linearization of the various yield conditions is required, introducing additional complexity. For this reason a concave contact formulation will be adopted in this paper. (Note also that, due to their comparative simplicity, concave contact formulations have also been widely embraced by the developers of 'physics engines', which can robustly analyse the dynamic response of large numbers of rigid blocks with contact and friction [8,9].)

Another important aspect in the computational limit analysis of rigid block assemblages relates to the modelling of plastic flow at contact interfaces. Either an associative or non-associative 'flow rule' can be assumed. However, an associative flow rule leads to normal displacement (dilatancy) accompanying sliding along a frictional contact interface. In contrast, when modelling 'Coulomb friction', zero dilatancy should be assumed, indicating that flow is non-associative. Generally the response of a real dry frictional joint involves dilatancy somewhere between these two extreme cases [10]. In practice the assumed flow rule can have a significant influence on the computed collapse load factor, and the collapse load factor computed using an associative friction model will represent an upper bound on the factor calculated using a non-associative friction model [2]. In the interests of safety, it is therefore generally preferable to adopt a Coulomb friction (i.e. non-associative) model when calculating the collapse load factor.

If the interactions at contact interfaces are characterised by non-associative flow, the problem of modelling collapse of a discrete rigid block system has been shown to lead to a non-symmetric mixed complementarity problem (MCP), with linear or non-linear constraints [11]. This problem does not have a unique solution, and the issue of how best to find suitable solutions for engineering purposes arises.

Different solution methods specifically for three-dimensional rigid block assemblages and non-associative frictional contacts are described in the literature. For example, Baggio and Trovalusci [12,13] and Orduña and Lourenço [14] have proposed convex contact formulations, using non-linear programming (NLP) to obtain solutions. Orduña also presented a concave formulation [15], proposing the use of a 'load path following' procedure previously developed and applied in a convex contact formulation to obtain solutions. These formulations appear capable of providing good predictions of the collapse load factors and corresponding failure modes for masonry structures. However, the solution of the MCP problems by nonlinear programming generally involves long CPU-times when applied to rigid block assemblages when large numbers of blocks are involved, whether applied to 2D or 3D problems [14,15].

As an alternative to directly solving the mixed complementarity problem (MCP) arising from the assumption of non-associative behaviour, a simpler procedure based on iterative solution of associative problems can alternatively be used, potentially saving significant CPU time. This is because when associative friction is assumed the MCP is symmetric, and can be uncoupled into dual linear or conic programming problems, either of which can be solved efficiently using interior point methods [16].

Livesley [5] was amongst the first to develop a computational model for three-dimensional masonry block problems, adopting a concave contact formulation involving four contact points per interface, an associative friction model and using linear programming to obtain solutions. As with his formulation for 2D rigid block problems [4], he proposed that the apparently unrealistic dilation which occurs at sliding contacts in the collapse mechanism should be corrected in a single post-processing step. This can be achieved by replacing the pyramidal failure surface used for sliding with a prismatic failure surface, 'associated' to the no-dilatancy behaviour of classical Coulomb sliding friction. Although Livesley was aware