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Introduction to m–m processes ☆

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Abstract

In this paper, we introduce a new type of nonlinear model, called the min–max model, and analyze its properties for a pair of series. The stability conditions of this system are given for a nonlinearly integrated bivariate series. Under these stability conditions, the difference between the two series exhibits threshold-type nonlinearity. It is possible to construct a threshold error correction model from the min–max processes. Neglected nonlinearity tests are applied, both to the univariate series and to the bivariate system, in order to detect nonlinearity, and it turns out that the tests using the bivariate series have better power. We apply the min–max model to U.S. Treasury bills and commercial paper interest rates. The spread of these interest rates shows threshold-type nonlinearity, and this model outperforms a linear model in terms of its predictability for out-of-sample data.

JEL classification: C22; C32

Keywords: Min–max process; Nonlinear error correction model; Neglected nonlinearity; Threshold; Interest rate spread

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1. Introduction

A new class of nonlinear models is introduced, with particular emphasis on bivariate processes. This process has some theoretical interest, since univariate series contain weak evidence of nonlinearity, but this evidence becomes strong when a bivariate system is considered. A bivariate system can be linearly cointegrated, but with a nonlinear error-correction model. A bivariate process is generated by means of the following equations:

$$x_{t+1} = \max(\alpha x_t + a, \beta y_t + b) + \varepsilon_{x,t+1}, \tag{1}$$

$$y_{t+1} = \min(\gamma x_t + c, \delta y_t + d) + \varepsilon_{y,t+1}, \tag{2}$$

where $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ are i.i.d. with variances σ_x^2 and σ_y^2, respectively. Although the max–min pair will normally be used, pairs such as max–max or min–min are equally acceptable, thus giving rise to the title m–m. All such pairs are related using rule A3, provided in the appendix. Thus, the min in Eq. (2) could be replaced by $-\max(\gamma^* x_t + c^*, \delta^* y_t + d^*)$ where $\gamma^* = -\gamma$, $c^* = -c$, $\delta^* = -\delta$, and $d^* = -d$. It should be noted that linear equations can be obtained by using the values $b = -\infty$ and $c = +\infty$.

A form of particular interest is obtained when $\alpha = \beta = \gamma = \delta = 1$ and is called the "integrated m–m process", having

$$x_{t+1} = \max(x_t + a, y_t + b) + \varepsilon_{x,t+1}, \tag{3}$$

$$y_{t+1} = \min(x_t + c, y_t + d) + \varepsilon_{y,t+1}. \tag{4}$$

This is the bivariate version of the system discussed by Olsder and Delft (1991), but with stochastic terms added. To see how this system works in at least one case, let us suppose that $a<0$ and $d>0$. Without the y_t term in the max component of Eq. (3), x_t would be a random walk with downward drift, but the y_t term may nudge x_t toward a higher set of values, whereas in Eq. (4) the opposite is the case. Thus, the two series are closely intertwined, and the marginal processes, $\mathrm{E}[x_t|x_{t-j}, j>0]$ and $\mathrm{E}[y_t|y_{t-j}, j>0]$, are inclined to have quite different properties from those of the joint process.

These are examples of nonlinear processes, since the max function in Eq. (3) sometimes chooses $y_t + b$ and the min function of (4) sometimes chooses $x_t + c$. One may relate the nonlinear behavior to the level of z_t ($\equiv x_t - y_t$), which will be explained in detail in Section 3. An empirical example is given below, which shows that m–m models provide better fits on out-of-sample data than linear models. In addition, m–m models are shown to exhibit strictly nonlinear behavior, which linear models cannot emulate.

2. Equilibrium values

In this paper, a particular form of equilibrium will be considered. If there are no further stochastic shocks and if the process converges to constant values, so that